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By R. Preston McAfee and Daniel Vincent*

The theory of auctions represents in many ways the real success story of game theory applied to economics. Auction theory offers a strategic model that plausibly describes the precise extensive-form game that applies directly to a large class of economic activity. Robust and fairly general theoretical results can be obtained concerning existence, optimality, and comparative statics.[1] Despite these substantial theoretical gains, there remain major difficulties in taking this theory to the data.

The most obvious roadblock to testing auction theory is the heavy use made of unobservables in the theory. Bidders choose optimal bids based on signals that are not observed by the econometrician studying auction behavior. Although the theory maps the distribution of signals into a distribution of bids, the econometrician can only guess at the distribution of signals, and a statistical rejection of the implied distribution of bids is merely evidence that the econometrician guessed poorly. The lack of any a priori guidance about the appropriate distribution argues strongly for the development of testable implications that are *distribution-free*. With a notable exception,[2] auction theorists have by and large ignored this issue.

A second problem in received auction theory has been the neglect of entry conditions. With a handful of exceptions, auction theory tends to assume a known exogenous number of bidders. As anyone who has participated in an auction knows, this is clearly unrealistic. Entry is typically endogenous, and for many auctions, including the U.S. offshore oil auctions analyzed here, the actual number of participants is neither known *ex ante* nor is deterministic. Furthermore, bidders often incur costs of submitting bids which may be due either to the acquisition of their private information or to the actual preparation of the bid.

This paper generalizes the common-value auction model to allow for an endogenously determined number of bidders, with nature or some other whimsical agent playing a role. We construct a distribution-free test statistic in a general asymmetric-information model, which enables us to confront an economically important policy issue: how can the government determine whether it is setting an optimal reserve price? Using data to compute the optimal reserve prices has considerable empirical importance, for the U.S. government uses auctions to sell many public resources, notably oil and timber.

In the next section, we develop the generalized mineral-rights model, allowing for endogenous entry and stochastic participation, and derive the test on the reserve price. In the subsequent section, we present our analysis of the data for offshore oil leases. We show that there is compelling evidence to suggest that the government should set its reserve price substantially higher than current levels. We conclude with some remarks on the possibility of extend-

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[1] Modern auction theory began with William Vickrey's (1961) classic paper and enjoyed a revival starting in the late 1970's with the general treatments of Robert Wilson (1977) and especially Paul Milgrom and Robert Weber (1982). For a general survey, see McAfee and John McMillan (1987a). For more specialized surveys, see Milgrom (1985) and Wilson (1992).

[2] Kenneth Hendricks and Robert Porter, with colleagues, have produced several examples of analyses that are distribution-free and address significant economic concerns. In particular, the test of the winner's curse (Hendricks et al., 1987) and the test of the behavior of uninformed bidders (Hendricks et al., 1990) are good examples.

ing this approach to compute the optimal reserve price.

I. The Generalized Mineral-Rights Model

Suppose there are a maximum of N possible bidders. Nature chooses a subset $A = \{a_1,\dots,a_n\}$ to be *potential* bidders, with probability $q_n / \binom{N}{n}$, that is, nature chooses a number n of potential bidders with probability q_n and then chooses a subset of the possible bidders to be the potential bidders at random. Let $\bar{q} = \Sigma_{n=0}^{N} n q_n$ be the expected number of bidders. Nature's chosen subset of potential bidders will then choose to obtain a signal x_i about the value of the tract at cost s with probability ρ in complete ignorance of nature's choice, beyond their own selection by nature. Conditional on learning a signal, these *active* bidders will bid provided their signal x_i is greater than the *effective reserve price*, R. An active bidder with a signal exceeding R is called an *actual* bidder. The true value of the object is the common value v, and the signals of active bidders are, conditional on the true value v, independently distributed with cumulative distribution function $F(x_i|v)$ and density $f(x_i|v)$. The unconditional density of v is g, and we let E_v refer to expectation over v. Define $p(v|x)$ to be the conditional density of v given x. The seller is presumed to hold a first-price sealed-bid auction with reserve price, or minimum bid, r. The actual bidders with the highest bid obtains the object at a price equal to his bid. The common value v decomposes into two components; revenues (w) minus costs (c). The seller values an unsold tract at $\sigma(v)$ and collects a royalty rate α on revenues of tracts that sell, that is, the winning bidder with a bid of b obtains $(1-\alpha)w - c - b$. We ignore moral-hazard effects. In the auctions to be examined, $\alpha = 1/6$.[3]

Consider an actual bidder with signal x, who submits a bid b. Let other bidders use the increasing bidding function B. The bidder's expected profits are:[4]

$$\Pi(b,x) = \sum_{n=1}^{N} \frac{nq_n}{\bar{q}} \int [(1-\alpha)w - c - b]$$

$$\times \{1 - \rho[1 - F(B^{-1}(b)|v)]\}^{n-1}$$

$$\times p(v|x)\, dv.$$

An equilibrium bidding function B maximizes Π over b at $b = B(x)$, with endpoint condition $B(R) = r$. We let $\pi(x) = \Pi(B(x), x)$. There are two equilibrium entry conditions that determine R and ρ, at least when the expected number of bidders is sufficiently great that bidders use a mixed participation strategy $\rho \in (0,1)$. The first is that choosing to bid is optimal:

$$\pi(R) = 0.$$

The second condition is that choosing to take a signal is also optimal:

$$s = \int_R^{\infty} \pi(x) E_v f(x|v)\, dx.$$

The following result provides two characterizations of the seller's revenue in this environment.

[3]In offshore-oil auctions, royalties apply to revenue and not costs, and thus the profits of the winning bidder are $(1-\alpha)w - c$. Fixing a bidder, we assume that the density of the highest of other bidders' signals, conditional on $(1-\alpha)w - c$, satisfies the monotone-likelihood-ratio property with respect to $(1-\alpha)w - c$. This guarantees existence of a monotone equilibrium bidding function (see Milgrom and Weber, 1982).

[4]It is demonstrated in McAfee and McMillan (1987b) that the probability of n bidders, conditional on a given bidder's inclusion, is $nq_n/\bar{q}$. Note that our bidder, bidding b, wins the auction if his competitors either do not become active, with probability $1-\rho$, or do become active but have signals less than $B^{-1}(b)$. We assume that there is no bid b so that uninformed bidders will find it profitable to bid.

LEMMA 1: *The seller's rents are the net gains from trade, which are*:

$$(1)\quad \Psi = E_v\left[\sum_{n=1}^{N} q_n \int_R^{\infty} (v-\sigma)n \times \{1-\rho[1-F(x|v)]\}^{n-1} \times \rho f(x|v)\,dx\right] - \bar{q}\rho s$$

$$(2)\quad = E_v\left[\left(1-\sum_{n=0}^{N} q_n \times \{1-\rho[1-F(R|v)]\}^{n}\right) \times (v-\sigma)\right] - \bar{q}\rho s.$$

Remark 1: Equation (1) shows that the seller earns the value of the object for sale minus the bidders' signal acquisition costs. Since endogenous entry implies that bidder profits are, in the random-participation equilibrium, equal to the costs of bidder participation, the seller obtains the social surplus associated with trade.

The next result is the main theorem of this section and provides a distribution-free test for establishing whether ρ is too large or too small. The proof employs equation (2). It is useful to define the random variable κ, which is a function of the number of bidders n, conditional on $n \geq 1$, by

$$(3)\quad \kappa = \begin{cases} B(x) + \alpha w - \sigma & n = 1 \\ \max B(x_i) + c & n \geq 2 \\ \quad -(1-\alpha)w & \end{cases}$$

THEOREM 2:

$$\frac{\partial \Psi}{\partial \rho} \gtreqless 0 \quad \textit{as} \quad E[\kappa | n \geq 1] \gtreqless 0.$$

That is, the auction is attracting inefficiently few bidders whenever net revenues exceed the values of tracts that attracted two or more bidders. Moreover,

$$(4)\quad \frac{\partial \Psi}{\partial R} \gtreqless 0 \quad \textit{as} \quad E_v\left[(\sigma - v) \times \sum_{n=1}^{N} n q_n \{1-\rho[1-F(R|v)]\}^{n-1} \times f(R|v)\right] \gtreqless 0.$$

That is, R is too low if the seller's expected value for the tract exceeds the expected value of the tract conditional on the highest bid being r.

Remark 2: The important aspect of Theorem 2 is that the test for too few or too many bidders is (asymptotically) distribution-free; that is, it depends only on the observables given in equations (3), and not on the distribution F, the density of the common value g, or the selection probabilities q_n.

There is a simple intuition behind Theorem 2. Since all bidders value the tract equally *ex post*, obtaining more than one bidder reduces the social surplus, relative to obtaining one bidder. Note that $E[(1-\alpha)w - c - B(x)]$ is the expected profits of a bidder and therefore equals the participation cost. Thus, gaining an extra bidder costs $E[(1-\alpha)w - c - B(x)]$. A gain is made only in the instance when this obtains a bidder when there would not be one otherwise: the gain is $E[v-\sigma] = E[w - c - \sigma]$. There is no gain when $n \geq 2$; this yields κ, a gain of $w - c - \sigma - [(1-\alpha)w - c - B(x)]$ with one bidder, and a loss of $(1-\alpha)w - c - B(x)$ with two or more bidders.

II. Econometric Analysis of Outer-Continental-Shelf "Wildcat" Auctions

Hendricks and Porter provided us with data on outer-continental-shelf (OCS) wildcat[5] oil-lease auctions, and the estimates that they made of the *ex post* revenues and costs for the tracts.[6] We considered only the 1,264 auctions prior to 1972, because the expectation of world oil prices is reasonably thought to be constant prior to 1972 (see Hendricks et al. [1987], who also explain the process of calculating the *ex post* value estimates) and because the production of most of these wells is complete. Moreover, joint ventures (see the analysis by Hendricks and Porter [1992]) were rare prior to 1972. These auctions operate with an announced reserve \$15 per acre (generally tracts are 5,000 acres, yielding a \$75,000 reserve). However, bids as high as \$1,673,045 were rejected for unspecified reasons in 89, or approximately 7 percent, of the auctions. We exclude the tracts that failed to sell. Auctions with zero actual bidders are not observed. The number of bidders ranges from one to 18 with a mean of four.

Both the participation probability $\rho[1-F(R|v)]$ and the distribution of winning bids,

$$\sum_{n=0}^{\infty} q_n\{1-\rho[1-F(B^{-1}(b)|v)]\}^n,$$

are functions of the common value v. To control for this, we divide the tracts into ten categories based on the estimated *ex post* value, numbering the categories from 1 to 10 with higher values in higher categories. We treat tracts in a given category as being identical. That is, we treat the common value v as a discrete variable v_i taking on ten values corresponding to the means of the values in the categories. This leads to some errors in taking the model to the data. However, the test given in Theorem 2 depends on this approximation only through the estimation of the government's value for unsold tracts.

Table 1 offers some summary statistics on the data that document the discussion in the text. There are some items worth noting. Affiliation, in this model, would imply that the distribution of the number of bidders and the distribution of bids should increase in the first-order stochastic-dominance sense as the value v_i increases. There is substantial evidence that both of these in fact increase. From Table 1, we see that mean winning bids and the mean number of bidders tend to increase in v_i, for v_i nonnegative.[7] Moreover, there is compelling evidence, not reported here, that the distributions of the number of bidders and of the bids shift to the right across the categories with nonnegative values.

Participation, in this model, appears to follow a geometric distribution. That is, the number of participants n appears to have the probability $(1-q_i)q_i^n$, where the parameter q_i varies across categories. The estimated q_i is expected to vary across categories, since it is composed of an exogenous move by nature and a probability $\rho F(R|v_i)$. While the former is independent of category, the latter certainly is not. The geometric distribution is a special case of the negative binomial, which is commonly used to model stochastic numbers of participants. Although we will make no use of the actual distribution below, we report this here because it is potentially important for extend-

[5] Wildcat auctions are distinguished from *drainage* tracts, in which a bidder owns an adjacent tract and thereby has better information about the tract value than other bidders. In wildcat tracts, bidders' information tends to be more symmetric, and therefore the symmetric model is more reasonably applied.

[6] All monetary variables are deflated to constant 1972 dollars.

[7] We presume that the reason that firms lost money in the first five categories is because they had quite strong priors that there was oil present (i.e., had a high signal). This points to the fact that we have not controlled for the endogeneity of the drilling decision. This would appear to be a relatively small factor in the data, given the maximum loss. However, it is probably important to control for the endogenous drilling decision in future work.

TABLE 1—SUMMARY OF DATA BY CATEGORIES

Value range						
Min	Max	NT	$E\pi$	E[WB]	n	MT
−17	−2.5	66	−5.0	5.6	5.4	5
−2.5	−1.5	95	−2.1	5.4	5.7	8
−1.5	−1	113	−1.3	3.9	4.6	10
−1	−0.5	191	−0.8	2.7	3.6	27
−0.5	−0	132	−0.4	1.7	3.1	33
−0	+0	272	0.0	0.6	2.0	130
+0	4	95	0.6	3.6	4.6	18
4	15	72	6.5	4.1	4.9	5
15	40	66	19.0	4.8	5.7	4
40	304	73	66.8	7.8	6.1	3
−17	304	1,175	4.9	3.2	4.0	243

Notes: Values and bids are in millions of dollars. The first two columns give the value range defining the category, with the last row being a summary of all the data. The third column (NT) is the number of tracts in that category. The fourth column ($E\pi$) gives the average profits on the tracts in that category. The fifth column (E[WB]) provides the mean winning bid, and the sixth column (n) provides the average number of bidders. Finally the last column provides the number of tracts in the category with winning bids less than $250,000.

ing the present analysis to computing optimal royalty rates as well as reserve prices.

Remark 3: The data contain no information about bidders with signals below the effective reserve R, since these active bidders do not participate and therefore cannot guide us in calculating the effect of lowering the reserve r.

The effect of raising the reserve price decomposes into two effects, by (2), via R and ρ:

$$\frac{\partial \Psi}{\partial r} = \frac{d\Psi}{dR}\frac{\partial R}{\partial r} + \frac{d\Psi}{d\rho}\frac{\partial \rho}{\partial r}. \tag{5}$$

In what follows, we assume that, as the reserve is increased, R increases and ρ decreases. Thus, to sign (5), we need to compute the effect of increasing R and ρ. Both of these effects depend on the value of not selling, σ.

If one assumes that oil is a resource in fixed supply and satisfies the Hotelling rule, then the value of the tract is constant in present-value terms. This presents a problem, because then the seller should set a reserve equal to the expected value of the tract, which in turn equals the expected maximum bid, which of course must exceed the reserve. However, we know from the work of Paul Romer and Hiroo Sasaki (1985) that, in the presence of technological improvements, prices of resources in fixed supply need not rise. Assume instead that oil prices are expected to be constant, a reasonable assumption prior to 1972. Then, the value of a marginal tract can be approximated as follows. If the tract fails to sell, we presume that the government attempts to sell the tract in the future, discounting earnings by δ. This leads to the equation $\sigma(v) = \delta E[\max B(x_i) + \alpha w | v]$. To make this operational, we somewhat arbitrarily define marginal tracts as those that attracted maximum bids of no more than $250,000. Assuming that these tracts are representative of their categories, we can estimate σ by a weighted average of the mean royalties plus winning bids in each category, weighted by the likelihood that the marginal tracts fell into that category. The average winning bids and the frequency of marginal tracts are given in Table 1. Discounting this estimate by $\delta = 0.737$, chosen because it represents 3 percent per year for a decade,[8] we obtain an estimate of σ for marginal tracts of $1,702,101. It is important to realize that this procedure gives an estimate of the seller's value of tracts that might fail to sell if the reserve is raised, since only marginal tracts are considered. That is, we are estimating the value *conditional on the information that all current bidders estimated low values for the tract*.

Equation (4) demonstrates that R is too low if the government's value of marginal

[8]The methodology employed here assumes that, if the government fails to sell a tract and then attempts to resell it in a decade, the potential bidders would have new draws for their signals and would employ the same equilibrium bidding function as is observed in the extant data. Therefore, it is appropriate to compute the empirical distribution of values for marginal tracts and to use this distribution to weight the observed average winning bids, discounted by the time to resale.

tracts, σ, exceeds the *ex post* value of marginal tracts. Continuing to define a marginal tract by a maximum bid below \$250,000, the average *ex post* value of those tracts is \$948,455. There is a source of error in this number, as some of these tracts are still producing, but future production has been estimated. The \$750,000 gap between the value to the government of marginal tracts and the *ex post* value of the tracts is strong evidence that R is too low.

We are now in a position to see that the reserve price, r, should be increased. We expect that increasing the reserve price will increase the effective reserve, or cutoff value for actual bidders, R, and will decrease the participation probability ρ. The first-order effect on Ψ from increasing R is estimated to be positive, since the government's value, σ, exceeds the average value of the marginal tracts. The value of reducing the participation probability ρ is found by computing $E\kappa$. We computed $E\kappa$ by setting the government's value σ_i for tracts in category i to equal the average winning bid in category i plus the share of the value, discounted by $\delta = 0.737$. This yields an average for κ of $-1{,}548{,}864$, with a t statistic of -3.01, a clear indication that decreasing the participation probability will increase the government's expected revenue.[9]

This leads to a persuasive argument that reserve prices should have been much higher before 1972, probably over \$1,000,000.[10] All calculations were in constant 1972 dollars, and this translates into over 3 million 1992 dollars, an increase of 40-fold over the current reserve. We expect that the increase in oil prices and price fluctuations since 1972 will increase the desirability of raising the current reserve price. Although this may seem to be an unacceptably high reserve price, it should be noted that the government has rejected bids in excess of \$1 million in 1972 dollars on 60 different tracts. Thus, we are suggesting that the government implement as a rule a policy that has up to now been rather whimsically employed.[11]

III. Conclusion

This paper extends the generalized mineral-rights model to allow for stochastic endogenous participation. Since this implies that the seller extracts the entire social surplus, the standard result in auctions with a fixed number of bidders—that the seller desires to distort away from efficiency by posting an inefficiently high reserve price—does not extend to this model. We argue that taking asymmetric information models to the data requires designing distribution-free test statistics, and we demonstrate that such statistics may be available by providing a test of when the reserve price is too low. In addition, we offer a methodology for computing the value to the government of a marginal tract, which is unobservable.

This analysis can be used in virtually any common-value context, provided that a measure of *ex post* value is available. One important attribute of the analysis is that the test statistic κ and the estimate of the government's value depend only on the *ex post* value and the winning bid. Thus, similar tests should be available for oral auctions, in which the data permit observation of winning bids but not losing bids. In particular, the present analysis should be directly applicable to U.S. Department of the Interior timber auctions, which account

[9]The value of κ, divided by the number of bidders, provides an estimate of the cost s of becoming informed. The value obtained is consistent with the estimate of seismic survey costs reported by Hendricks and Porter (1992).

[10]In order to increase $E[v|R]$ by \$750,000 (to \$1,700,000), it is necessary to increase r by more than \$750,000, since at the higher value of R an actual bidder is more likely to win, even holding ρ constant, and ρ will fall, further increasing profits at the increased level of R. Thus $dE[v|R]/dr$ is typically less than unity, and a larger increase than \$750,000 is necessary just to satisfy (4). In addition, it is desirable to increase r still further, as there is a *direct* advantage to decreasing ρ. All told, \$1,000,000 seems to be a reasonable estimate.

[11]Hendricks et al. (1990) offer evidence that the stochastic employment of a reserve price is unrelated to the value of the tract.

for about half of all timber sold in the United States.

The model employed in the paper also allows for potentially more precise policy recommendations. Signals in this model are not in any directly meaningful units; that is, the model is essentially unchanged if we employ a monotonic transformation of the signals. A useful monotonic transformation is the bidding function itself. That is, given a distribution of signals $F(\cdot|v)$, consider the new distribution of signals $F(B^{-1}(\cdot)|v)$. With the transformed distribution, bidders optimally choose to bid their signal. This provides a nonparametric estimate of the distribution $F(\cdot|v_i)$ for each of the ten categories.[12] These distributions can be used to forecast the effect of increasing reserve prices and royalty rates by recomputing the equilibrium bidding functions when these parameters change. The computation of the optimal reserve price and royalty rates and the resulting expected revenue then becomes feasible.

[12]Of course, bidding one's signal works only in the base case; when we contemplate changing the reserve price, the equilibrium bidding function will cease to be the identity function.

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