HORIZONTAL MERGERS IN SPATIALLY DIFFERENTIATED NONCOOPERATIVE MARKETS*

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We present a Cournot–Nash model of horizontal mergers between firms that engage in spatial price discrimination. The model extends the analysis of such mergers as presented in the US Department of Justice Merger Guidelines. Rather than conclude the evaluation of such a merger with an estimate of the post-merger HHI, as is done in the Merger Guidelines, our model yields an estimate of the increases in the equilibrium, post-merger delivered prices caused by the merger.

I. INTRODUCTION

The study of the equilibrium effects of horizontal mergers has recently attracted great attention. The subject was introduced by Salant, Switzer, and Reynolds [1983]. They use a homogeneous product, Cournot–Nash model with non-spatially differentiated firms. They show that if all firms have the same constant marginal costs, then most mergers between firms that play Cournot–Nash both pre- and post-merger are not profitable. Perry and Porter [1985] argue that mergers in constant average cost models are not well specified. A merger between two firms with different constant average costs results in the high-cost firm being shut down. Such shutdowns are almost never observed in real mergers. The sole gain to the low-cost firm from the merger is the elimination of the high-cost firm as a rival. Since the low-cost firm has no capacity constraints, it has no use for the assets of the high-cost firm. Intuitively, a merged firm should be “bigger” than either of the two pre-merger firms because it combines the assets of the two firms.

Perry and Porter use a homogeneous product, Cournot–Nash, quadratic-cost model with non-spatially differentiated firms and show that many horizontal mergers are profitable. Farrell and Shapiro [1990] and McAfee and Williams [1992] use this model to consider the welfare effects of horizontal mergers. Daughety [1990] shows that even in the Salant, Switzer, and Reynolds model where all firms have the same constant marginal cost, some mergers are profitable if the behavior of the merged firm changes from pre-merger Cournot to post-merger Stackelberg. Levin [1990] assumes firms have differing but constant marginal costs and non-merging firms play

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Cournot both pre- and post-merger. He shows under certain demand and cost conditions that, regardless of the behavior of the merged firm, a profitable merger of firms that have no more than 50% of the pre-merger total industry output will raise welfare.

One feature missing from the extant literature on the equilibrium effects of horizontal mergers is how to delineate the relevant market. Since the models assume a homogeneous product and non-spatially differentiated firms, the problem of how to separate firms in the market from firms outside of the market is assumed away.\(^1\) Although these assumptions are commonly made in oligopoly models, they lead to problems when considering horizontal mergers. The primary problem is that discussion of the "market" in which firms compete becomes problematic. What does it mean to refer to the "market" when the geographic extent of the market is a single point and all firms in the market produce a homogeneous good? In order to evaluate the competitive effects of a merger, one must have an algorithm for identifying the relevant set of firms in the market. The relevant market bifurcates the set of all firms into two subsets: those firms that can be safely ignored when evaluating the competitive effects of a proposed merger and those firms that must be considered. The existing literature on the equilibrium effects of horizontal mergers offers no guidance on how to identify the market.

Market definition has received considerable attention in the antitrust literature, without coherent conclusions until recently. The situation improved in 1982 with the introduction by the US Department of Justice of a revised set of merger guidelines. A major contribution of the 1982 Merger Guidelines was its use of an integrated approach to merger analysis, including market definition. In this regard, the unifying theme of the 1982 Merger Guidelines was to prohibit mergers that significantly enhance the ability of firms to collude, either expressly or tacitly,\(^2\) and the test for market definition was directly tied to that behavioral assumption. The 1982 Merger Guidelines introduced a two-step framework under which the market is first defined based on the "hypothetical monopolist" paradigm and then market concentration is assessed by the Hirschman–Herfindahl Index (HHI) and used to estimate whether the merger will significantly increase the likelihood of tacit or overt collusion. The Merger Guidelines were revised in 1984 without altering the approach to assessing the effects of mergers.

In 1992, the Merger Guidelines were revised again and issued jointly by the Department of Justice and the Federal Trade Commission. The new revision addresses not only collusion, but "unilateral" conduct that appears to encompass noncooperative behavior, such as Cournot–Nash behavior. The consideration of noncooperative behavior is a welcome change, since limiting

\(^1\) Deneckere and Davidson [1985] consider the profitability of mergers in a differentiated products Bertrand model with non-spatially differentiated firms.

\(^2\) See: Merger Guidelines § 1; Baxter [1985] and Werden [1987].
the behavioral assumption to collusion presents both empirical and practical difficulties.\(^3\)

We consider the instance in which firms producing a homogeneous good engage in spatial price discrimination.\(^4\) Rather than focus on collusion, we consider a model in which firms play Cournot–Nash both pre-merger and post-merger. Our paper extends the Merger Guidelines' analysis of horizontal mergers involving price discriminating firms in several respects. First, we use an equilibrium model of firm behavior, rather than the non-equilibrium HHI criteria in the Guidelines. Second, rather than conclude the analysis with an estimate of the post-merger HHI, our model yields an estimate of the increases in the equilibrium, post-merger delivered prices caused by the merger. Finally, we show that the analysis of mergers in noncooperative markets where firms engage in price discrimination should combine the market definition and market concentration/performance studies into one step, in contrast to the two-step procedure in the Merger Guidelines.

We use a Cournot–Nash model with firms located at different points on a plane. At each location in the plane, firms have differing constant marginal (delivered) costs. An alternative spatial model of noncooperative behavior can be based on Bertrand–Nash behavior. In spatial models with homogeneous products, Bertrand–Nash behavior leads to non-overlapping sales territories (Anderson and Neven [1990]). In contrast, Cournot–Nash spatial models with homogeneous products lead to overlapping sales territories. Since

\(^3\) With respect to explicit collusion, there is no extant model of explicit collusion to apply. On empirical grounds, very few mergers lead to explicit collusion (Joyce [1987]). Concerning tacit collusion, there are models in which firms engage in repeated games and, for sufficiently low discount rates, achieve monopoly prices (Kreps, [1990]). A problem with dynamic oligopoly models is that they admit more collusion than is observed. In the extreme, full-information Bertrand models with a five percent real discount rate and a three day response lag supports the monopoly price with 2,493 identical firms. In contrast, there is empirical evidence that the Cournot–Nash model approximates the behavior of some markets. For example, in a recent study of duopoly airline city-pair markets, Brander and Zhang [1990], p. 567 concluded: "...we find that the Cournot–Nash model seems much more consistent with the data than the Bertrand or cartel model." Similarly, in a cross-section study of 445 4-digit SIC manufacturing industries, (Gisser [1986], p. 763) concludes the empirical results lend "...support to the hypothesis that the [four largest firms in each industry] are not engaged in collusive agreements, or if they do, engage in 'secret' price cutting, such that their behavior is roughly approximated by the Cournot–Nash model." In a study of firms engaging in spatial price discrimination in the cement industry, McBride [1983] concludes that the spatial Cournot model applies to the regional markets in his study. Using experimental data, Fouraker and Siegel [1963] find that in markets where firms do not know their rivals' profits with certainty, the best predictor of the market price is the Cournot–Nash price. Again using experimental data, Friedman and Hoggart [1980] find that if (1) firms do not have perfect knowledge of their rivals' cost functions or (2) firms do not have symmetric profit functions, then as summarized by Plott [1982], p. 1517: "In the duopoly markets, significant (but less than perfect) cooperation occurs but, with an increase in the number of firms, it vanishes almost completely and the Cournot model is very accurate by comparison."

\(^4\) If firms price discriminate, the 1992 Merger Guidelines state that "...the Agency will consider additional geographic markets consisting of particular locations of buyers in which a hypothetical monopolist could profitably and separately impose at least a 'small but significant and nontransitory' increase in price." 1992 Merger Guidelines § 1.22.
overlapping sales territories are empirically observed, the Cournot–Nash assumption appears more appropriate (Phlips [1983]).

By construction, the model applies to markets in which firms (approximately) behave as Cournot–Nash competitors both pre- and post-merger. An interesting, but unsolved, problem is how to delineate the relevant market and evaluate the likely equilibrium post-merger price changes when firms engage in repeated noncooperative games. In such circumstances, the equilibrium price may fluctuate dynamically depending on the state of play. For sufficiently low discount rates, firms engaged in repeated noncooperative games can achieve the monopoly price in equilibrium (Kreps [1990]).

The model is presented in Section II. Conclusions are discussed in the last section.

II. THE MODEL

There are \( n \) firms and at each point in \( \mathbb{R}^2 \) the firms play Cournot–Nash. Firms are indexed by \( i, i \in \{1, \ldots, n\} \), producing quantity \( q_i \geq 0 \) (which depends on location, but this dependence is temporarily suppressed). Output is \( Q = \sum_{i=1}^{n} q_i \). Demand is characterized by the inverse demand function \( p \). We consider three different regularity assumptions about demand:

\[
\begin{align*}
(R1) & \quad (\forall Q) 2p'(Q) + Qp''(Q) < 0 \\
(R2) & \quad (\forall Q) p'(Q) + Qp''(Q) < 0 \\
(R3) & \quad (\forall Q) p''(Q) \geq 0
\end{align*}
\]

The first regularity condition \((R1)\) is equivalent to decreasing marginal revenue. The second condition \((R2)\) is the standard regularity condition and causes reaction functions to slope downward. The third condition \((R3)\) requires demand to be convex. We denote the elasticity of demand by

\[
\varepsilon = -\frac{p(Q)}{Qp'(Q)}
\]

At a fixed location, let the firms’ marginal costs be constant and given by \( c_1 \leq c_2 \leq \ldots \leq c_n \). This is tantamount to assuming that each location represents a small part of a firm’s total output. A firm earns profits of \( \pi_i = (p(Q) - c_i)q_i \). Firms selling a positive quantity satisfy

\[
0 = q_i p'(Q) + p(Q) - c_i
\]

The profit function \( \pi_i \) is concave in \( q_i \) provided \((R1)\) is satisfied. Suppose that \( k \) firms are “active” at a fixed location, i.e. they sell \( q_i > 0 \). Then \( Q \) is given by

\[
0 = Qp'(Q) + kp(Q) - \sum_{i=1}^{k} c_i
\]

An equilibrium \((Q, k)\) satisfies \((1)\) for \( i \leq k \), equation \((2)\), and
\( c_k \leq p(Q) \leq c_{k+1} \). A straightforward argument, available from Michael Williams, shows that, given (R1), the equilibrium is unique. Moreover, the equilibrium has the interesting property that the value of \( k \) maximizes the quantity \( Q \) and hence minimizes price, subject to (2). In particular, for linear demand \( p(Q) = 1 - Q \), the equilibrium price is

\[
(3) \quad \min_{A \subseteq \{1, \ldots, n\}} \frac{1 + \sum_{i \in A} c_i}{1 + |A|}
\]

where \(|A|\) is the number of elements in \( A \). Let \( s_i \) be the market share \( q_i/Q \) of firm \( l \). From equation (1), we obtain, for \( i \leq k \),

\[
(4) \quad \frac{p(Q) - c_i}{p(Q)} = \left( \frac{q_i}{Q} \right) \left( -\frac{Qp'(Q)}{p(Q)} \right) = \frac{s_i}{\epsilon}
\]

Now consider merging firms \( i \) and \( j \), each of which may have multiple plants. At any fixed location, the merged firm will use the plant with the lowest delivered cost. By (1), if \( s_i < s_j \), then \( c_i > c_j \), and the merged firm has cost \( c_i \) at the given location. Note as well that, unlike Salant, Switzer, and Reynolds [1983], the merged firm does not shut down any plants unless a plant has strictly higher delivered cost at every location than one of the merged firm’s other plants. Thus, a merger does result in a “bigger” firm because the merged firm combines the plants of the two firms.

Antitrust enforcement is generally concerned with the effects on output prices resulting from merger. The following theorem establishes bounds on the price effects—bounds that are generally quite “tight,” at least if there are several players in the market. Let \( \frac{\Delta p}{p} \) be the percentage price change resulting from a merger at a given location.

**Theorem 1.** Consider a merger between firms \( i \) and \( j \), with \( s_i \leq s_j \) at a given location, and let \( k \) be the number of active firms. Then

\[
(5) \quad (R2) \text{ implies } \frac{\Delta p}{p} \leq \frac{s_i \log \left( \frac{k}{k-1} \right)}{\epsilon \left[ 1 - \log \left( \frac{k}{k-1} \right) \right]} \in \left( \frac{s_i}{\epsilon(k-1)}, \frac{s_i}{\epsilon(k-2)} \right)
\]

\[
(6) \quad (R1) \text{ implies } \frac{\Delta p}{p} \leq \frac{s_i \log \left( \frac{k-1}{k-2} \right)}{\epsilon \left[ 1 - \log \left( \frac{k-1}{k-2} \right) \right]} \in \left( \frac{s_i}{\epsilon(k-2)}, \frac{s_i}{\epsilon(k-3)} \right)
\]

If merger does not prompt entry and (R3) holds, then

\[
(7) \quad \frac{\Delta p}{p} \geq \frac{s_i \log \left( \frac{k+1}{k} \right)}{\epsilon} \geq \frac{s_i}{\epsilon(k+1)}
\]
If merger may prompt entry and (R3) holds, then

\[ \frac{\Delta p}{p} \geq \frac{s_i}{\varepsilon(k + 1)} \]

Remark 1. A reasonable approximation for the percentage price change is \( s_i/\varepsilon k \). This works exactly for the case of linear demand without potential entry. It should be noted that the lower bounds depend on the convexity of demand, a property which, while reasonable in some contexts, is not a standard assumption.

Remark 2. The data requirements for computing the bounds on the price changes are quite modest. In particular, the analyst must know (i) the quantity shares \( s_i \) of the merging firms for each relevant location,\(^5\) but not the shares of the other firms,\(^6\) (ii) the number of firms with positive sales at the location under consideration, and (iii) the pre-merger elasticity of demand.\(^7\)

Theorem 1 suggests a notion of relevant geographic markets based on noncooperative behavior. Choose an acceptable price change, which for concreteness we take to be 5%. Define the relevant geographic market to be that area where the merger will probably cause prices to increase by at least 5%. Thus, using the heuristic of Remark 1, the relevant area is the set of locations in which the lesser market share of the two merging firms exceeds \( \frac{\varepsilon k}{20} \), where \( \varepsilon \) is the pre-merger elasticity of demand and \( k \) is the number of firms selling at the location under consideration. Unlike the notion of relevant geographic markets provided by the 1984 Merger Guidelines, a larger area in the present definition is cause for more concern, because it means that a larger number of consumers will be adversely affected.

III. CONCLUSIONS

We have presented a Cournot–Nash model of horizontal mergers between firms that engage in spatial price discrimination. The data requirements of the model are quite modest, involving variables that are observed in the course of most merger investigations. We are not, however, advocating that the model

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\(^5\) Since it is generally difficult to estimate elasticities and market shares at each point, the model can be approximated by letting relatively small areas be locations. Such areas might be cities, counties, or even states, depending on the application and availability of data.

\(^6\) This is a major advantage over Hirschmann–Herfindahl indices, because the aggregate sales at a given location may be known from industry reports, even when the sales of firms not party to the merger are unknown.

\(^7\) The elasticity of demand is generally estimated in order to utilize the analysis of the current merger guidelines, because the question of whether a hypothetical monopolist would raise price requires information on the elasticity of demand. Thus, the data requirements for the present study are strictly less than those associated with implementing the Merger Guidelines.
be used for policy purposes at this time because the empirical validity of the model remains to be shown.

One of the insights of the model we find of interest is the unification of the market definition and market concentration steps in predicting the equilibrium post-merger delivered prices. In contrast, the Guidelines use a two-step framework suitable for analyzing increased likelihood of collusion that may result from a merger. However, if the firms engaging in spatial price discrimination play Cournot–Nash both pre-merger and post-merger, then the market definition and market structure/performance steps must be performed at the same time. The model unifies these steps, by defining markets based on predicted price increases resulting from merger, which in turn depends on the extent of the competition prevailing before and after the merger. That is, the model suggests defining relevant geographic markets by areas affected by mergers, and not by a criterion independent of the nature of competition prevailing in the region. By focusing directly on the firms active at each location, the corresponding demand and cost conditions, and using the Cournot–Nash behavioral assumption, we estimate the equilibrium, post-merger (delivered) prices, thereby identifying the region likely to be adversely affected.

Future research along these lines must confront two problems. First, the analysis must be extended to delineate product markets as well as geographic markets. Spatial competition may provide an acceptable model of differentiated product markets, but the practical problem of identifying the “location” of a firm’s product in product space must be resolved.

Perhaps more importantly, additional empirical research is required to identify a reasonable class of models to confront proposed mergers. It seems likely that no single model will be appropriate for all industries, but observable characteristics of products may serve as a guide for model selection. Identifying the relevant characteristics for matching industries and models appears to be a major research agenda. Finally, the approach embodied in Theorem 1, that of identifying heuristics for the effects of mergers, could be important in merger analysis.

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APPENDIX

Proof of Theorem 1. Implicitvly define $Q(x)$ by

(A1) \[ 0 = Qp'(Q) + (k - 1)p(Q) - \sum_{i \neq i}^{k} c_i + \alpha(p(Q) - c_i) \]

Note that $p(Q(1))$ is the pre-merger price, and $p(Q(0))$ is the post-merger price, provided that no entry occurs, so that $p(Q(0))$ is an upper bound on the post-merger price. Thus, the percentage change in price does not exceed

\[
\frac{\Delta p}{p} = \frac{p(Q(0)) - p(Q(1))}{p(Q(1))}
\]

\[
= \frac{-1}{p(Q(1))} \int_{0}^{1} p'(Q(x))Q'(x)\,dx
\]

\[
= \frac{1}{p(Q(1))} \int_{0}^{1} p'(Q(x)) \frac{p(Q(x)) - c_i}{(k + \alpha)p'(Q(x)) + Q(x)p''(Q(x))} \, dx
\]

\[
= \frac{1}{p(Q(1))} \int_{0}^{1} \frac{p(Q(x)) - c_i}{k + \alpha + \frac{Q(x)p''(Q(x))}{p'(Q(x))}} \, dx
\]

\[
\leq \frac{1}{p(Q(1))} \int_{0}^{1} \frac{p(Q(x)) - c_i}{k + \alpha + A} \, dx
\]

where $A = \{-1, -2\}$ if (R2) holds

\[
\leq \frac{p(Q(0)) - c_i}{p(Q(1))} \int_{0}^{1} \frac{1}{k + A + \alpha} \, dx
\]

\[
= \frac{p(Q(0)) - p(Q(1)) + p(Q(1)) - c_i}{p(Q(1))} \log \left( \frac{k + A + 1}{k + A} \right)
\]

\[
= \left( \frac{\Delta p}{p} + \frac{s_i}{\varepsilon} \right) \log \left( \frac{k + A + 1}{k + A} \right)
\]

thus, $\frac{\Delta p}{p} \leq \frac{s_i \log \left( \frac{k + A + 1}{k + A} \right)}{\varepsilon \left[ 1 - \log \left( \frac{k + A + 1}{k + A} \right) \right]}$
A straightforward argument establishes that, for $n \geq 2$,

$$\frac{1}{n-1} \leq \log \left( \frac{n}{n-1} \right) \leq \frac{1}{n-2}$$

With the definition of $A$, this gives (5) and (6). To establish the lower bounds, we now presume that (R3) holds. Similar to the previous argument, if no entry occurs we have

$$\frac{\Delta p}{p} = \frac{1}{p(Q(1))} \int_0^1 \frac{p(Q(\alpha)) - c_i}{k + \alpha} \frac{Q(\alpha) p''(Q(\alpha))}{p'(Q(\alpha))} d\alpha$$

$$\geq \frac{1}{p(Q(1))} \int_0^1 \frac{p(Q(\alpha)) - c_i}{k + \alpha} d\alpha$$

$$\geq \frac{p(Q(1)) - c_i}{p(Q(1))} \int_0^1 \frac{1}{k + \alpha} d\alpha = \frac{s_i}{\varepsilon} \log \left( \frac{k+1}{k} \right)$$

(by (4))

This holds provided that the price change does not provoke entry, and gives (7). If the price change provokes entry, at most one firm, with cost $c_N$ exceeding the pre-merger price (otherwise it would already have been active) enters. The appropriate function $Q(\alpha)$ is now defined by

\[(A2) \quad 0 = Qp'\langle Q \rangle + kp\langle Q \rangle - \sum_{i \neq i}^k c_i - \alpha c_i - (1 - \alpha)c_N\]

Thus, $p(Q(0))$ is the post-merger price, and $p(Q(1))$ is the pre-merger price, and $c_N \geq p(Q(1))$.

$$\frac{\Delta p}{p} = \frac{p(Q(0)) - p(Q(1))}{p(Q(1))}$$

$$= -\frac{1}{p(Q(1))} \int_0^1 Q'(\alpha)Q'(\alpha) d\alpha$$

$$= \frac{1}{p(Q(1))} \int_0^1 \frac{Q'(\alpha)(c_N - c_i)}{(k+1)p'(Q(\alpha)) + Q(\alpha)p''(Q(\alpha))} d\alpha$$

(differentiating (A2))

$$\geq \frac{c_N - c_i}{p(Q(1))} \int_0^1 \frac{1}{k+1} d\alpha \geq \frac{p(Q(1)) - c_i}{p(Q(1))(k+1)} = \frac{s_i}{\varepsilon(k+1)}$$

(by (4))

This gives (8).

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