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The non-existence of pairwise-proof equilibrium

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Abstract

Private bilateral contracting between a supplier and competing customers admits multiple equilibria. We show that requiring equilibrium to be 'pairwise proof' – immune to bilateral deviations by the supplier and any customer – can imply non-existence of equilibrium in 'normal' environments.

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1. Introduction

Several authors have recently analyzed situations where one firm is a supplier to two or more firms and contracts with each *bilaterally* and *privately* (Cremer and Riordan, 1987; Horn and Wolinsky, 1988; Hart and Tirole, 1990; O'Brien and Shaffer, 1992; and McAfee and Schwartz, 1994). In Cremer and Riordan, a supplier faces several non-competing buyers but transactions are interdependent because the supplier's marginal cost is not constant. In the other papers, transactions are interdependent because the buying firms compete in a downstream market. The rationale for studying private bilateral contracting – despite the interdependence among transactions – is that third parties may be unable to observe others' dealings or verify these to a court, or that making one contract optimally contingent on others may be quite difficult due to sheer complexity and bounded rationality.

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Characterizing the outcome of such bilateral vertical contracting is problematic. Private bilateral contracting generally admits multiple Perfect-Bayesian-Nash Equilibria (PBNE), because changing the offer to one firm could lead that firm to revise its beliefs about what the supplier is offering to others; the latitude in belief revisions off the equilibrium path in turn can support a wide range of equilibria. The above authors identify a unique outcome by requiring equilibrium contracts to be *pairwise proof*: immune to bilateral deviations by the supplier and any customer, holding constant all other contracts. Pairwise proofness is not implied by PBNE, since the same seller is simultaneously contracting with several buyers.

Cremer and Riordan, and O'Brien and Shaffer impose pairwise proofness directly, by invoking it as a primitive solution concept – their *contract equilibrium*.[1] Hart and Tirole, and McAfee and Schwartz do not invoke the contract-equilibrium concept but obtain pairwise proofness as a necessary condition for PBNE by imposing a restriction on firms' off-equilibrium beliefs, *passive beliefs* (Hart and Tirole's 'market-by-market bargaining'). Passive beliefs require that a firm receiving an offer different from what it expected in the candidate equilibrium continues to believe that others receive their equilibrium offers. One justification of passive beliefs is that each firm interprets a deviation by the supplier as a tremble and assumes trembles to be uncorrelated (say, because the supplier appoints a different agent to deal with each firm). Thus, all the above authors directly or indirectly impose pairwise proofness on the equilibrium outcome.

Pairwise proofness and the closely related passive-beliefs restriction seem reasonable in some settings. The purpose of this paper, however, is to caution against indiscriminate use of the pairwise-proofness criterion. It can be an overly strong requirement that leads to frequent non-existence of pure-strategy PBNE in 'normal' environments. Our example involves a monopolist supplier offering two-part tariffs to several firms, which then learn each others' costs and compete Cournot downstream. Section 2 of the paper describes the environment, and Section 3 presents the non-existence result. Section 4 discusses why an equilibrium does exist in the other papers cited above, and in the related work of Vickers (1985).

2. The environment

Consider an input supplier with constant marginal cost, $z > 0$, and no fixed cost. The supplier faces $n \geq 2$ potential downstream firms that use the input to produce homogeneous products. We confine attention to contracting in two-part tariffs. A two-part tariff offered to firm i is a pair (r_i, f_i) where f_i is a fixed fee and r_i is the marginal price per unit of the input. Idle firms can be viewed as accepting the contract $(\infty, 0)$ and producing zero output.

The n downstream firms have total cost functions $C_i(q_i) = r_i q_i$. That is, a firm has constant marginal cost equal to the input price it faces. Downstream competition among the active firms will be Cournot. Aggregate output is denoted $Q = \sum q_i$, and inverse demand is $p(Q)$. We

[1] Horn and Wolinsky's 'simultaneous bargaining' has the same flavor. They confine attention to linear pricing (the other authors allow fixed fees) and assume that the linear price between the supplier and each firm is determined through Nash bargaining taking the other price as given.

make a standard assumption that inverse demand $p(Q)$ is such that industry marginal revenue everywhere decreases in output:

$$\forall Q: 2p'(Q) + Qp''(Q) < 0 \,. \tag{1}$$

For simplicity, suppose the supplier makes take-it-or-leave-it offers; as explained later, our non-existence result would extend if fixed fees were instead determined through bilateral bargaining.

The timing of moves is:

Stage 1 (offers): The supplier privately makes a set of offers, one for each firm: $\{r_i, f_i\}$, $i = 1, \ldots, n$.

Stage 2 (acceptances): Firms accept or reject offers simultaneously. Accepting means paying the fixed fee.

Stage 3 (learning): Accepted contracts are learned by all, hence firms learn others' marginal costs.

Stage 4 (competition): Firms simultaneously (i) choose outputs; and (ii) purchase the corresponding amounts of the supplier's input.

The above timing reflects our assumption that contracting is bilateral and private (stages 1 and 2) and that the fixed fee charged to a firm cannot be made contingent on the terms offered to others (no renegotiation after stage 3). Given our costs and demand conditions, for any vector of input prices $\boldsymbol{r}' = (r'_1, \ldots, r'_n)$ *accepted* in stage 2 and learned in stage 3, there is a unique Cournot equilibrium in stage 4. It is harmless to consider only accepted contracts as $(\infty, 0)$ is trivially accepted. Let $q_i(\boldsymbol{r}')$ denote the indirect input-demand function and $\pi_i(\boldsymbol{r}')$ denote firm i's indirect equilibrium-profit function, where $\pi_i = [p(Q) - r'_i]q_i$, revenue minus variable costs. In equilibrium, all correctly expect prices $\boldsymbol{r}$. So in stage 2, it is a (weakly) dominant strategy for firm i to accept the supplier's contract offer if $f_i \leq \pi_i)(\boldsymbol{r})$, since firm i's profit will be at least $\pi_i(\boldsymbol{r})$ if rivals' reject their contract offers. Thus, in equilibrium the supplier (in stage 1) sets $f_i = \pi_i(\boldsymbol{r})$ to each firm i. Denote by u_i the sum of the supplier's net revenue from all input sales and of firm's i's profit:

$$u_i(\boldsymbol{r}) = \sum_{j=1}^{n} (r_j - z)q_j(\boldsymbol{r}) + \pi_i(\boldsymbol{r}) \,. \tag{2}$$

Observe that u_i only depends on the marginal prices, $\boldsymbol{r}$, and not on the fixed fees.

Definition. A *contract equilibrium* is a set of contracts $\{r_k, f_k\}$, $k = 1, \ldots, n$ satisfying:

$$V \equiv \sum_{k=1}^{n} [(r_k - z)q_k(\boldsymbol{r}) + \pi_k(\boldsymbol{r})] \geq 0 \text{ (monopolist's individual rationality)} \,, \tag{3}$$

and

$$u_i(r_i, \boldsymbol{r}_{-i}) \geq u_i(r'_i, \boldsymbol{r}_{-i}) \text{ (\textit{pairwise proofness})} \,, \tag{4}$$

for any r'_i and all $i = 1, \ldots, n$ where the first entry denotes the input price to firm i and $\boldsymbol{r}_{-i}$ denotes the vector of input prices charged to others.

The individual rationality (IR) condition for each firm i, $\pi_i(r) \geq f_i$, is already embedded in condition (3) with equality since the supplier sets $f_i = \pi_i(r)$. Condition (3) states that the supplier's profit V, inclusive of all fixed fees, must be non-negative. Conditions (4) states that, holding all other contracts fixed, the joint profit of the supplier and any firm i cannot be increased by altering their contract.

Imposing passive beliefs on firms in stage 2 of our game implies that if a PBNE exists, it must be pairwise proof; otherwise, a bilaterally-profitable deviation would be accepted given the firm's belief that it alone was offered the deviation. A contract equilibrium must be robust only against bilateral deviations. In contrast, PBNE under passive beliefs must also be immune to deviations that the supplier might offer to two or more firms and that those firms would accept under passive beliefs. To facilitate comparison with the work of Cremer and Riordan, and O'Brien and Shaffer, we prove a theorem on the non-existence problems created by requirements (3) and (4) alone, then discuss additional problems posed by multilateral deviations.

3. Non-existence

The theorem below, proved in the appendix, first characterizes how far below the supplier's marginal cost, z, input prices must be to satisfy pairwise-proofness – conditions (4). This characterization applies only if the resulting input prices are non-negative, which in turn will hold if the supplier's cost z is 'sufficiently high'. Offering negative input prices can never be profitable for the supplier, as a firm would demand an infinite input quantity. The theorem then provides conditions under which these pairwise-proof input prices imply an output price below z, thereby violating the supplier's IR, condition (3).

Theorem. If the supplier's marginal cost z is sufficiently high, in a contract equilibrium all firms must be active and produce equal outputs Q/n satisfying:

$$p(Q) - z = \frac{Q}{n}\left[p'(Q)(n-2) + p''(Q)\frac{n-1}{n}Q\right].$$

This implies that $[p(Q) - z] < 0$ if

(i) $n \geq 2$ and $p''(Q) < 0$; or

(ii) $n \geq 3$ and $p'(Q) + Qp''(Q) < 0$; or

(iii) $n \geq 4$.

In each case, the supplier's profit would be negative, so there exists no pure-strategy contract equilibrium.

The result is understood as follows. Under our assumptions, pairwise proofness implies that equilibrium must be symmetric: all n firms receive equal input prices, hence produce equal outputs. Consider the candidate equilibrium where the supplier prices at marginal cost to all: $r = z$. The input price accepted by any firm i is observed by others before downstream competition occurs, so by cutting r_i the supplier can induce firms $j \neq i$ to shrink their planned outputs, and can collect firm i's expected profit increase by raising i's fixed fee initially.

Starting from $r = z$, the change in *bilateral* profit u_i from cutting price only to firm i – the relevant test for pairwise proofness – comes entirely from this strategic (rent-shifting) effect: $\partial u_i(z)/\partial r_i = q_i p'(Q)\, \partial Q_{-i}/\partial r_i$, where Q_{-i} is the Cournot-equilibrium output of all firms excluding i. Constant costs and assumption (1) imply $\partial Q_{-i}/\partial r_i > 0$, so $\partial u_i(z)/\partial r_i < 0$. Therefore, $r = z$ is not pairwise proof. Nor is any $r > z$: cutting r to any firm would then increase bilateral profit because of the strategic effect *and* of increased input sales (starting from equal input prices, $\partial Q/\partial r_i < 0$). Thus, pairwise proofness requires $r < z$.

If all firms face $r < z$, raising r_i has three effects on u_i: (i) the 'integrated structure' gains from cutting firm i's sales, as $r_i < z$; (ii) firm i's revenue falls as rivals expand outputs (strategic effect); and (iii) the supplier loses as i's rivals expand their *input purchases*. The third effect is crucial. Even if r is so low that output price is below cost ($p < z$), raising r_i can increase the supplier's losses: it would induce higher input purchases by firms $j \neq i$, whose fixed fees are held constant when evaluating a bilateral deviation with i. Thus, pairwise proofness can dictate $p < z$. But the supplier's profit is then negative, violating IR.

The theorem establishes non-existence by imposing only IR and no profitable *bilateral* deviations. A contract equilibrium considers only bilateral deviations, but PBNE under passive beliefs must be immune also to certain multilateral deviations: offers which the supplier can profitably make to *several* downstream firms and which firms would accept under passive beliefs. This added requirement has some bite. Observe that a firm i accepts any contract $(r_i, 0)$, since the firm pays no fixed fee and buys inputs only after learning rivals' costs. So under passive beliefs, the supplier's PBNE profit must (weakly) exceed not only 0, to satisfy IR, but also what the supplier can earn if it offers any set $\{r_i, 0\}_i$. Consider linear demand and two downstream firms ($n = 2$). The expression in the theorem, derived from pairwise proofness, implies $p = z$, hence zero profit. The contract equilibrium is then $(r^c < z, f^c > 0)$. But a PBNE under passive beliefs does not exist: deviating to $(r > z, f = 0)$— is (i) profitable, and (ii) violates pairwise proofness.

4. Related work

Several features explain why non-existence of equilibrium can arise in our model but not in related work: (a) the supplier offers *two-part tariffs*; (b) it sells to *several* firms that *compete* Cournot downstream; and (c) downstream competition occurs after each firm learns all contract terms (ex post observability).

Horn and Wolinsky's game is similar to ours except for (a): their contracts feature only linear input prices, determined through simultaneous Nash bargaining between the supplier and each firm.[2] If contracts included linear prices and fixed fees, the Nash-bargaining price in each transaction would maximize the joint surplus of the supplier and that firm (the fixed fee would merely divide the surplus). Equilibrium marginal prices would then have to be pairwise proof as here, causing similar non-existence problems.

[2] Horn and Wolinsky show that the duopolists can lose from merging, because merging would weaken their bargaining power versus the supplier enough to offset the gain from monopolizing the output market.

Cremer and Riordan, and Vickers (1985) differ from our model regarding (b). In Cremer and Riordan, the supplier's customers do not compete downstream. A contract equilibrium then exists and entails marginal-cost pricing. Intuitively, if customers do not compete, overall efficiency for the contracting group and bilateral maximization both require marginal-cost pricing.[3] In Vickers (1985), there is Cournot competition among dealers, but each dealer is its manufacturer's *sole* agent. Given that downstream competition is Cournot (more precisely, if downstream instruments are strategic substitutes), each manufacturer prices its input below marginal cost ($r < z$), for the same strategic (downstream rent-shifting) reason as in our model. But our supplier sells to *several* competing firms. This creates a stronger incentive to cut r: once $r < z$, cutting r further to any one firm benefits our supplier by reducing the below-cost input sales to the supplier's other customers. This effect is absent in Vicker's model, explaining why an equilibrium exists.

Hart and Tirole, O'Brien and Shaffer, and McAfee and Schwartz analyze a game that differs from the one here regarding (c): firms never observe others' contracts. Given *unobservability* and passive beliefs (or contract equilibrium), the outcome under quite general conditions about downstream competition or downstream production technology is for the supplier to set price equal to marginal cost to all firms.[4] The difference caused by unobservability is that the supplier can no longer strategically influence the downstream competition by changing the price to any one firm. Also, under passive beliefs a firm expects that a received deviation contract is not offered to others, hence will not change others' actions. In its contracting with each firm the supplier therefore acts as if the two are integrated and face a residual downstream demand function that is invariant to the input price. Pairwise maximization then involves setting input price equal to the supplier's marginal cost.[5]

In conclusion, the problem of private bilateral contracting when contracts affect third parties is both economically important and relatively unexplored. Invoking passive beliefs (hence pairwise proofness) yields marginal-cost pricing under unobservability, and in that environment this outcome has some intuitive appeal. But, as demonstrated, passive beliefs lead to frequent non-existence of equilibrium if contracts become known prior to downstream competition. A possible reconciliation is to argue that passive beliefs are less plausible under such ex post observability than under unobservability, since firms are less likely to perceive unexpected offers (deviations from the expected equilibrium) as independent trembles by the

[3] Cremer and Riordan introduce asymmetric information about customers' types, and also allow for more complex hierarchical structures. Their main concern is to show that bilateral contracts can be designed to elicit parties' private information and attain the efficient solution even in such richer environments.

[4] McAfee and Schwartz consider only two-part tariffs. Hart and Tirole, and O'Brien and Shaffer allow general contracts, where a firm's charge is a possibly non-linear function of its input order, and their outcome is the same as would be induced by marginal-cost pricing under two-part tariffs. O'Brien and Shaffer's outcome is the unique contract equilibrium. Hart and Tirole, and McAfee and Schwartz obtain this outcome as the unique PBNE under the added restriction of passive beliefs.

[5] The 'pairwise maximization' logic relies also on the assumption that firms order their inputs before learning their sales. If, instead, firms competed in prices and ordered inputs only after learning their sales, the monopolist's revenues from firm B generally would depend on the input price offered to firm A. Therefore, the price-equals-marginal-cost result will not hold, in general, when inputs are purchased after sales. O'Brien and Shaffer show that this result nevertheless obtains under certain conditions.

supplier.[6] But we do not offer this as a general resolution. The goal of this paper is simply to point out some pitfalls of what might at first seem a natural requirement of equilibrium under private multilateral contracting, namely, that it be immune against bilaterally profitable deviations. This restriction is far from innocuous.

Appendix

Proof of the theorem. An active firm chooses an equilibrium quantity q_i satisfying

$$0 = q_i p'(Q) + p(Q) - r_i \,. \tag{A1}$$

If firms $1, \ldots, m$ are active, $m \leq n$, then

$$0 = Qp'(Q) + mp(Q) - \sum_{i=1}^{m} r_i \,. \tag{A2}$$

The right-hand side of (A2) is monotonically decreasing in Q, showing that for any number of active firms the Cournot equilibrium is unique.

Differentiating (A2) with respect to r_i yields:

$$\frac{\partial Q}{\partial r_i} = [(m+1)p'(Q) + Qp''(Q)]^{-1} < 0 \,, \tag{A3}$$

the inequality following from assumption (1) in the text. Differentiating (A1) with respect to r_i and substituting from (A3) gives

$$\frac{\partial q_j}{\partial r_i} = \begin{cases} \left[m + \dfrac{p''(Q)}{p'(Q)}(Q - q_i)\right]\dfrac{\partial Q}{\partial r_i} < 0 \,, & i = j \leq m \,, \\ -\left[1 + \dfrac{p''(Q)}{p'(Q)} q_j\right]\dfrac{\partial Q}{\partial r_i} \,, & i \neq j \,, \quad i, j \leq m \,, \end{cases} \tag{A4}$$

where the inequality follows from (1) and (A3).

The supplier chooses r_i to maximize

$$u_i = \sum_{j \neq i} (r_j - z)q_j + (p(Q) - z)q_i \,. \tag{A5}$$

Suppose the first m firms are active. Then for $i \leq m$,

[6] For example, the assumption that trembles are independent is more appealing when the monopolist appoints different agents rather than one to deal with the different firms. Appointing different agents, in turn, makes more sense for the monopolist when its profits from the various transactions are independent (as they are under unobservability) than when they are interdependent (as under ex post observability).

$$\begin{aligned}\frac{\partial u_i}{\partial r_i} &= \sum_{j\neq i}^{m} (r_j - z)\frac{\partial q_j}{\partial r_i} + q_i p'(Q)\frac{\partial Q}{\partial r_i} + (p(Q) - z)\frac{\partial q_i}{\partial r_i} \\ &= \sum_{j\neq i} (r_j - z)\frac{\partial q_j}{\partial r_i} + q_i p'(Q)\frac{\partial Q}{\partial r_i} + (p(Q) - z)\left[\frac{\partial Q}{\partial r_i} - \sum_{j\neq i}\frac{\partial q_j}{\partial r_i}\right] \\ &= \sum_{j\neq i} q_j p'(Q)\frac{\partial q_j}{\partial r_i} + [p(Q) + q_i p'(Q) - z]\frac{\partial Q}{\partial r_i} \\ &= \frac{\partial Q}{\partial r_i}\left[-p'(Q)\sum_{j\neq i} q_j\left[1 + \frac{p''(Q)}{p'(Q)} q_i\right] + p(Q) + q_i p'(Q) - z\right] \\ &= \frac{\partial Q}{\partial r_i}\left[p(Q) - z - p'(Q)[Q - 2q_i] - p''(Q)\sum_{j\neq i} q_j^2\right]. \end{aligned} \tag{A6}$$

Expression (A5) is not generally concave in r_i, although it is for linear demand. Nevertheless, if p is twice continuously differentiable, u_i will be continuously differentiable in r_i and first-order conditions will hold. For the supplier to be in equilibrium we have the Kuhn–Tucker conditions:

$$\frac{\partial u_i}{\partial r_i} = 0\,, \quad \text{if } q_i > 0 \quad \text{and} \quad \frac{\partial u_i}{\partial r_i} \geq 0\,, \quad \text{if } q_i = 0\,. \tag{A7}$$

It is straightforward to show that all active firms have equal equilibrium outputs and that all n firms will be active: $q_i = Q/n > 0$, $i = 1, \ldots, n$ (details available on request). Thus, (A3), (A6) and (A7) imply:

$$p(Q) - z = \frac{Q}{n}\left[p'(Q)(n-2) + p''(Q)\frac{n-1}{n}Q\right]. \tag{A8}$$

Case (i) in the theorem follows immediately from (A8).

Case (ii) follows by noting, for $n \geq 3$, $(n-1)/[n(n-2)] \leq 1$, and thus, for $p'' \geq 0$,

$$p'(Q)(n-2) + p''(Q)\frac{n-1}{n}Q \leq (n-2)[p'(Q) + Qp''(Q)]\,.$$

Case (iii) follows from noting, for $n \geq 4$, $(n-1)/[n(n-2)] < 1/2$, and using assumption (1). Q.E.D.

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