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Split-award contracts with investment ☆

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ABSTRACT

This paper studies procurement contracts where a buyer can either divide full production among multiple suppliers or award the entire production to a single supplier. We examine the effect of using multiple suppliers on investment incentives. In a framework of generalized second-price auctions with pre-auction investment, we show that the optimality of split-award depends on the socially efficient number of firms at the investment stage. When that number is greater than one, sole-sourcing is buyer-optimal. When that number is one, split-award lowers the buyer procurement cost.

1. Introduction

The practice of employing a contractual format that allows for a split-award is widely observed. In Japan's telecommunications industry, as reported in Fransman (1995), "Competition between the suppliers is not of the 'winner-take-all' variety. Rather, it involves controlled competition insofar as, contingent on reasonable performance as judged and monitored by NTT, each supplier can expect to receive a sizable share of NTT's order." China Mobile, the world's largest mobile carrier, regularly conducts several rounds of supplier tournaments in each year, where large chunks of equipment and mobile handset contracts are divided among a few vendors. The practice is also very common in the private sector. Wal-mart's vitamin business adopts a multiple-source model, relying on several of its vendors in China for supply contracts. Similarly, companies such as Sun and HP that use online auctions to procure products worth hundreds of millions of dollars frequently opt for multiple sourcing, as documented in Tunca and Wu (2009).

The issue of sole-sourcing versus split-award has been extensively researched. Some, for example Beltramo (1983), argues that split-buy competitions often increase costs to the government since they fail to achieve efficiency in production. Others, for example Gansler and Lucyshyn (2009), argue that with careful planning, this barrier (of achieving efficiency in production) can be overcome. The latter cites example of Lockheed Martin which was able to reduce the D-5 Sea-Launched-Ballistic-Missile production rate from 60 a year to 12 a year, while reducing the cost at the same time. They also report that the so-called "Great Engine War" to supply the F-16 and F-15 aircrafts demonstrated that the introduction of a second production source resulted in a dramatic improvement in engine reliability, higher performance and lower unit costs from both suppliers.

The existing theoretical analysis largely supports the dominance of sole-sourcing over multiple-sourcing (or split-award) in a variety of settings, as we will detail below. This paper considers the effect of split-award on the incentives to invest by analyzing an auction with cost-reducing investment. We show that the optimality of split-award from the buyer's perspective depends on the log-concavity or log-convexity of the distribution of realized costs, as a function of investment. When the distribution is log-convex, which implies investment exhibits decreasing returns to scale, it is socially efficient to have more than one firm at the investment stage, and sole-sourcing also minimizes buyer procurement cost in that case. When that distribution is log-concave and investment exhibits increasing returns to scale, it is socially efficient to have one firm at the investment stage. The auction implements the outcome efficiently; however that outcome is not optimal for the buyer from a cost minimization perspective. With one strong and one very weak bidder at the auction stage, the procurement cost is maximized, rather than minimized. Using a split-award mechanism, such that the second supplier invests as well, disciplines the strong bidder at the bidding stage, and lowers the

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buyer's procurement cost in comparison to sole-sourcing. Therefore, we arrive at our main insight that the optimality of split-award depends on the socially efficient number of firms at the investment stage. When that number is greater than one, sole-sourcing is buyer-optimal. When one firm investing in cost reduction is efficient, split-award lowers the buyer procurement cost.

Auctions generate efficient investment incentives. Thus, when efficient investment involves several firms (a diseconomy of scale in investment), an auction creates efficient investment and in the process creates competition, so the auction price is attractive to the buyer. In contrast, when efficient investment involves a single firm, the auction still produces efficient investment and necessarily that involves little competition, and thus prices are unattractive to the buyer. By employing a split-award, the buyer induces investment by a second firm, and the effect on price from the resulting competition is advantageous to the buyer. Thus, we conclude that dual-sourcing is advantageous for the buyer when only one firm should invest from a social perspective, and conversely. That is, we establish a somewhat paradoxical result that dual sourcing is buyer-optimal when investment by a single firm is socially optimal.

Our analysis uses generalized second price auction (GSP) for its appealing simplicity and the clean intuition of the analysis. Besides its wide application in the online market, the GSP may proxy bargaining, which is extensively used in private sector contracting.

The paper is organized as follows. We start by relating our contribution to three literatures: split-award auctions, auctions preceded with investments and generalized second-price auctions. Section 2 presents a simple model of investment with deterministic outcome. Section 3 analyzes the case when investment determines the distribution of marginal costs. Section 4 unifies the results of Sections 2 and 3 under log-concavity and examines the robustness of the main insights in several directions. Concluding remarks are in Section 5. All proofs and technical details can be found in the Appendix.

1.1. Related literature

There is a sizable literature studying optimal procurement practices where the primary concern is a buyer's strategic choice between single sourcing and multiple sourcing. The analysis of split-award auctions started with Wilson (1979), which analyzes share auctions where bidders receive fractional shares of the item at a price that equates demand and supply of shares and he shows that share auctions generally decrease seller revenue in comparison to unit auctions where the item is awarded to the highest bidder.[1] In the procurement setting, Anton and Yao (1989) analyze a model where suppliers submit bids on each possible split of a contract. They show that split-award low-price auctions typically lead to higher prices for the buyer. Indeed, they conclude that "(the) equilibria (of the split-award auctions) have the property that the price to the buyer is maximized… Thus, not only do split-award auctions fail to promote competition, they effectively present bidders with an invitation for implicit price collusion". Perry and Sákovics (2003) analyze a sequential second-price auction where a larger primary contract and a smaller secondary contract are awarded, and show that if the number of suppliers is fixed, sole sourcing leads to a lower procurement cost. Inderst (2008) confirms the result that a monopolistic buyer conducting an auction strictly prefers single sourcing.

A number of papers then go on to develop arguments for when and why split-award auctions could still be beneficial.[2] Riordan and Sappington (1989) show how second sourcing reduces information rents in a dynamic setting. In their model, single-sourcing is optimal in the one-shot, static framework but may not be optimal in a series of procurements. Anton and Yao (1992) and Anton et al. (2010) extend the setup in Anton and Yao (1989) to allow for asymmetric information among the suppliers about each other's production cost. They prove the existence of split-award equilibria when a diseconomy of scale in production is present. Following Anton and Yao (1989), these papers analyze a game where suppliers submit bids both on sole-sourcing and a given split. This structure of the game prevents the suppliers from using bids for different splits strategically and leads to the coordination outcome of bids on the splits. Our analysis differs from theirs in two important ways. First, by allowing the buyer to choose the split and restricting the suppliers to submit one single bid on that split, the problem of unused bids, which potentially can be used strategically, is eliminated. Second, a diseconomy of scale in production is not plausible in many applications, including most applications where dual-sourcing is observed. Our analysis shows that a split-award can be optimal for the buyer in the absence of diseconomy of scale in production. The driving force, in our setting, is the provision of investment incentives to the suppliers so that both suppliers invest to lower their marginal costs of production, which in turn lower their bids in the bidding stage and hence the buyer's procurement cost becomes lower than that under winner-take-all auctions.[3]

Moldovanu and Sela (2001) find that in tournaments multiple prizes may be optimal if costs of contestant efforts are convex. They do not examine the possibility that contestants may invest to improve their abilities, which distinguishes this thread of literature. Alcalde and Dahm (2011) find that if the buyer can choose her budget constraint endogenously and suppliers' marginal costs are asymmetric and public information, using a first-price split auction that depends on submitted bids may lead to lower procurement cost than sole-sourcing. Our model endogenizes the asymmetry of marginal costs by analyzing the suppliers' investment behavior prior to auctions.

The choice of auction format affects bidders' investment incentives in a nontrivial way. The earliest paper is McAfee and McMillan (1987), which considers a situation where firms must invest a fixed cost to obtain a draw of their production cost. They show that standard auction formats lead to efficient investment and are buyer-optimal. King et al. (1992) and Arozamena and Cantillon (2004) show that while a second-price auction typically provides efficient investment incentives, suppliers tend to underinvest under first-price auctions. Bag (1997) shows that if the buyer can charge discriminatory entry fees, a second-price auction is efficient and is optimal for the buyer. Tan (1992) shows that first- and second-price auctions are revenue equivalent if investment technologies exhibit diminishing and constant returns to scale, and in Piccione and Tan (1996), it is shown that these auction formats are also efficient in those cases.[4] We depart from this literature by considering the optimal share splitting structure that minimizes a buyer's procurement cost, instead of focusing on sole-sourcing standard auctions.

Finally, our formulation of the bidding stage is related to the growing literature on generalized second-price auction and its wide applications in the online market, e.g. sponsored search auctions run by Overture (now part of Yahoo!) and Google. Representative works include Edelman and Ostrovsky (2007), Edelman et al. (2007), and Börgers et al. (2007). In these papers, the shares are exogenous, in contrast to

[1] Bernheim and Whinston (1986) analyze first-price menu auctions and show that they always achieve efficient allocation.

[2] From empirical perspective, Burnett and Kovacic (1989) report that in DoD's procurement program, guaranteeing a minimum share of production is particularly important when DoD wishes to induce a firm to bid against an established producer. Using a dataset on the missile system by the U.S. Defense Department, Lyon (2006) supports that dual sourcing indeed lowers government procurement costs significantly.

[3] Despite their strong conclusion about the optimality of sole-sourcing in complete information framework, Anton and Yao (1989) use a numerical example that foreshadows our general analysis to illustrate that pre-bid investment of the suppliers might render split-award optimal for the buyer.

[4] King et al. (1993) present a model where governments first invest in their infrastructure level, and then compete in auctions for a plant to be built in their region by a single firm. They show that the unique equilibrium of the game exhibits asymmetry in investment.

our analysis of the buyer-optimal share. Furthermore, these studies focus on bidders' strategic behaviors under GSP. The implication of our work for GSP analysis is that, if advertisers can invest in increasing the value of their ads, and the seller can manipulate placement to change the click share between advertisers, the seller would create a second position only when a single position is efficient due to investment effects. One way the seller could influence the click flow is through rotating the ads across users, which would fit our model, though not the standard GSP model with exogenous shares.

2. Investment with deterministic outcomes

Consider a buyer that must procure one unit of a certain good which is fully divisible. There are two potential suppliers, $i=1,2$. Their marginal costs of production $c_i\in[0,\omega]$ are determined by pre-auction investments.

In allocating the production contract, the buyer uses a generalized second-price split-award auction where each supplier submits a sealed price $b_i\in[0,\omega]$ for the contract. The supplier with the lower bid is awarded a fraction α of the production contract at a price equal to the higher bid. The supplier who has submitted the higher bid is awarded the fraction $1-\alpha$ of the contract at a price equal to the maximum cost ω.

Before bidding takes place, each supplier has an opportunity to make an investment to reduce his marginal cost. The technology can be either deterministic or stochastic, as we will detail in later sections. Investment is observable but not verifiable, and hence cannot be contracted upon. The time structure of the game is:[5]

1. The buyer chooses and announces α.
2. Suppliers simultaneously and independently choose investment, which determines marginal cost. A marginal cost c costs $g(c)$.
3. Bids are submitted simultaneously, and restricted to lie in $[0,\omega]$. The lower bidder is awarded the fraction α of the contract at a price equal to the higher bid. The higher bidder is awarded the fraction $1-\alpha$ of the contract at a price equal to ω. In the event of a tie, each bidder is assigned an equal probability of being the lower bidder.

The buyer's objective is to design α in such a way that her procurement cost is minimized. Note that for the buyer, procurement cost is maximized by an $\alpha\in\left[0,\frac{1}{2}\right]$ where the unique equilibrium at the bidding stage is $b_1=b_2=\omega$ and total procurement cost for the buyer is ω. In the following we can safely restrict our analysis to $\alpha\in\left(\frac{1}{2},1\right]$ where the buyer procurement cost may be lower than ω.

2.1. The bidding stage

For $\alpha\in\left(\frac{1}{2},1\right]$, we identify an equilibrium in (weakly) dominant strategies for GSP. We denote the marginal costs realized from the investment stage by c_L and c_H with $c_L\le c_H$, and denote b_i to be the bids.

Proposition 1. *The following strategies form an equilibrium in (weakly) dominant strategies at the bidding stage:*

$$b_i^*=\frac{1-\alpha}{\alpha}(\omega-c_i)+c_i,\quad \text{for}\quad i\in\{H,L\}. \tag{1}$$

In the case of two bidders, indifference between being the high or low is a weakly dominant strategy. This bid only depends on the bidders' own costs and hence forms an equilibrium whether or not there is asymmetric information about costs.

Proposition 1 implies that at the bidding stage, a supplier's bid is a strictly increasing function in its marginal cost. The outcome of the auction stage is that the low cost supplier is awarded the fraction α of the contract and the high cost supplier is awarded the fraction $1-\alpha$ of the contract. When $\alpha=1$, the allocation of contract is *ex post* efficient in that the supplier with lower marginal cost obtains the entire production contract.

2.2. Investment

In this section, we examine the impact of investment with deterministic outcomes on the buyer's procurement policy. If a supplier does not invest, his marginal cost equals ω. Achieving a marginal cost $c\le\omega$ requires an investment $g(c)$ which is assumed to be twice continuously differentiable and has the following natural properties:

$$g'(c)<0,\quad g''(c)>0\quad \text{for}\quad c<\omega, \tag{2}$$

$$g(\omega)=0,\quad g'(\omega)=0,\quad g'(0)<-1. \tag{3}$$

Condition (2) implies that a lower marginal cost has a higher fixed cost and the fixed cost is convex. In condition (3), $g'(\omega)=0$ implies that at $c=\omega$, it is costless for a supplier to invest marginally to reduce his cost below ω, which will insure a positive level of investment. $g'(0)<-1$ implies that the cost of achieving a marginal cost of 0 is higher than its value.

Whether costs are revealed before the auction takes place does not matter because of the weakly dominant strategy bidding as shown in Proposition 1.

Foreseeing that at the bidding stage the low cost supplier will become the low bidder and receive the fraction α of the contract at a price equal to the high bid, and the high cost supplier will be the high bidder and receive the fraction $1-\alpha$ of the production contract at price ω, the suppliers' expected payoffs at the investment stage are respectively

$$\pi_L(c_H,c_L)=\alpha(b_H^*-c_L)-g(c_L)=(1-\alpha)(\omega-c_H)+\alpha(c_H-c_L)-g(c_L); \tag{4}$$

$$\pi_H(c_H,c_L)=(1-\alpha)(\omega-c_H)-g(c_H). \tag{5}$$

Note that the unique maximizers of Eqs. (4) and (5) are given respectively by

$$-\alpha=g'(c_L^*),\quad \text{and}\quad -(1-\alpha)=g'(c_H^*). \tag{6}$$

In the next proposition, we state the equilibria at the investment stage.

Proposition 2. *The pure strategy equilibria at the investment stage are given by* (c_L^*,c_H^*) *and* (c_H^*,c_L^*).[6]

2.3. The buyer procurement policy

The buyer pays the equilibrium high bid b_H^* for the fraction α of the contract and ω for the remainder. Therefore, the buyer's total payment is:

$$m(\alpha)=\alpha b_H^*+(1-\alpha)\omega=2(1-\alpha)\omega+(2\alpha-1)c_H^*. \tag{7}$$

If $\alpha=1$, the buyer awards the entire contract to the low cost supplier at a price equal to c_H, and the low cost supplier gets a payoff equal to c_H-c_L, which is exactly the outcome of a standard Vickrey auction.

In the next theorem, we establish that the buyer's optimal choice of α is strictly smaller than 1, i.e. splitting the contract is strictly better than awarding the whole production to a single supplier.

[5] We discuss the impact of the buyer commitment power in Section 5.

[6] For a discussion on mixed strategy equilibria, please refer to Section 4.5.

Theorem 1. *Given the deterministic investment technology, splitting the contract between the two suppliers by choosing $\alpha<1$ is optimal for the buyer.*

This theorem is demonstrated by showing $\frac{\partial m}{\partial \alpha}|_{\alpha=1}>0$, and thus a slight reduction in α reduces buyer cost, starting at $\alpha=1$. A reduction in α has a zero first order effect on suppliers but a positive first order effect on the price paid to the low cost supplier. More technically, the objective of the buyer can be decomposed into two parts: the payment made to the high bidder $(1-\alpha)\omega$ and the payment made to the low bidder αb_H^*. When the suppliers' marginal costs are exogenous, it is optimal for the buyer to set $\alpha=1$ and award the entire contract to the low bidder. However, when marginal costs are endogenous, the choice of α affects the suppliers' payoffs from the bidding stage, and in turn their investment incentives, and finally the buyer's procurement cost. At $\alpha=1$, one supplier makes no investment and the other invests positively. While the high cost supplier is paid nothing, the low cost supplier is paid the second lowest cost, ω. This is the worst possible cost for the buyer. In contrast, for any α less than one but greater than 1/2, the total cost will be ω for the $1-\alpha$ portion, and the bid of the high cost supplier, which is less than ω, for the α portion. Thus it is always optimal for the buyer to split the award.

When $\alpha=1$, the contract is allocated to the low cost supplier, which is ex post efficient. Ex ante, only one supplier invests, which is also socially desirable given the investment technology. However, there is a tension between social efficiency and buyer's objective of procurement cost minimization. When α is equal to 1, at the bidding stage, the buyer faces a strong and a weak supplier, and the outcome is that the procurement cost is maximized. By guaranteeing a fraction of the contract to the weak bidder, that bidder also invests positively at the investment stage, and increases the competition at the bidding stage. Therefore, when investment equilibrium is asymmetric, the buyer can use split-award to discipline the strong bidder at the bidding stage.

Corollary 1. *When the equilibrium at the investment stage for $\alpha=1$ is asymmetric, i.e. one supplier invests positively and one supplier does not invest, split-award lowers the buyer's procurement cost in comparison to sole-sourcing.*

We close this section with an example.

Example 1. Suppose the investment technology is $g(c)=\delta(\omega-c)^2$, where $\delta>0$ and $\delta\omega>\frac{1}{2}$ to insure positive costs. $\pi_L(c_L,c_H)$ and $\pi_H(c_L,c_H)$ are uniquely maximized at $c_L^*=\omega-\frac{\alpha}{2\delta}$ and $c_H^*=\omega-\frac{1-\alpha}{2\delta}$. The buyer's expected procurement cost is $m(\alpha)=\omega-\frac{(2\alpha-1)(1-\alpha)}{2\delta}$, and the optimal α is $\frac{3}{4}$. Under sole-sourcing with $\alpha=1$, the buyer's procurement cost is equal to ω. Splitting the award optimally by $\alpha=\frac{3}{4}$ leads to a percentage cost saving of $\frac{1}{16\delta\omega}$, which can be as large as 12.5%, depending on the magnitude of δ and ω.

3. Investment with stochastic outcomes

In this section we investigate investment with stochastic outcomes, where investment determines the distribution of final marginal costs $c_i\in[0,\omega]$ for producing one unit of the good. This case is much more plausible in environments with research and development, where the outcomes are hard to predict. Realized costs c_i are drawn from cumulative distribution $H(c_i|x_i)$ with fixed support $[0,\omega]$, where x_i is the investment of firm i. $H(c|x)$ is assumed to have smooth density $h(c|x)$. We assume that for all $c\in(0,\omega)$:

$$H_2(c|x)\equiv\frac{\partial}{\partial x}H(c|x)>0, \tag{8}$$

$$H_{22}(c|x)\equiv\frac{\partial^2}{\partial x^2}H(c|x)\le 0. \tag{9}$$

An increase in investment decreases cost, in the first-order stochastic dominance sense, at a decreasing rate.[7] Further, we assume $H(c|0)=0$ for $c<\omega$, and $H(\omega|x)=1$ for all x.

Following Piccione and Tan (1996), we classify the investment technology by the project failure rate with respect to the level of investment. For all $c\in(0,\omega)$ and $x>0$, define

$$r(c,x)=\frac{H_2(c|x)}{1-H(c|x)}. \tag{10}$$

As developed by Piccione and Tan (1996), when $r(c,x)$ strictly decreases in x for any $c\in(0,\omega)$, the investment exhibits decreasing returns to scale; and if $r(c,x)$ is strictly increasing in x, investment exhibits increasing returns to scale. Furthermore, $r(c,x)$ decreases in x if and only if $1-H(c|x)$ is log-convex, and increases in x if and only if $1-H(c|x)$ is log-concave.

At the investment stage, the suppliers simultaneously and independently choose investment levels x. The distributions of marginal costs c_i are realized and observed. Suppliers draw their marginal costs from their respective distribution $H(c|x)$. The realized marginal costs are private information. Without loss of generality, the cost of investment is normalized to x.[8]

Recall that at the bidding stage there exists an equilibrium in (weakly) dominant strategies given by Eq. (1) in Proposition 1, and b_i^* is monotonically increasing in c_i. At the investment stage, the two suppliers simultaneously choose their investment levels which affect the payoffs at the bidding stage. By investing x_i, the distribution of marginal cost of supplier i is $H(c|x_i)$.

Lemma 1. *A supplier's expected payoff at the investment stage is given by*

$$\Pi_i(x_i,x_j)=\int_0^{\omega}H(c_i|x_i)(\alpha-(2\alpha-1)H(c_i|x_j))dc_i-x_i,\quad i=1,2. \tag{11}$$

The first and second order conditions of Eq. (11) are given respectively by:

$$\frac{\partial}{\partial x_i}\Pi_i(x_i,x_j)=\int_0^{\omega}H_2(c_i|x_i)(\alpha-(2\alpha-1)H(c_i|x_j))dc_i-1=0, \tag{12}$$

$$\frac{\partial^2}{\partial x_i^2}\Pi_i(x_i,x_j)=\int_0^{\omega}H_{22}(c_i|x_i)(\alpha-(2\alpha-1)H(c_i|x_j))dc_i\le 0. \tag{13}$$

The second order conditions hold globally by Eq. (9), so the first order conditions characterize the optimum. It will prove useful to split the analysis into the two cases of log-convexity and log-concavity.

Note that the investment stage is a Cournot competition with two firms choosing investment simultaneously, we can invoke the standard stability argument in selecting equilibrium when multiple equilibria exist. A stable equilibrium refers to a steady state such that starting from an arbitrary pair of investments that is sufficiently close to the steady state, in a dynamic adjustment interpretation of an equilibrium a la Cournot, the process always converges to that steady state. FOC (12) gives us the two best response curves of the two suppliers and the intersections of the two curves define the steady states of the game. If firm i's reaction curve is steeper than that of firm j at a steady state, the equilibrium is stable. Otherwise, it is unstable.[9]

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7 Inequality (9) is stronger than needed for the analysis, but it is the simplest condition to guarantee sufficiency of the first order conditions.

8 Using the convex investment cost of the previous section would not change the results. If the outcome distribution is $\hat{H}(c|\hat{x})$ and the investment costs $g(\hat{x})$, then letting $x=g(\hat{x})$ and $H(c|x)=\hat{H}(c|g^{-1}(\hat{x}))$ produces the present model.

9 For more on the stability issue, see, for example, Varian (1992, p.287–288), or Fudenberg and Tirole (1991, p.23–27).

3.1. Log-convex investment

When $1-H(c|x)$ is log-convex, we first show that at the investment stage, there exists a unique symmetric equilibrium which is stable.[10] Then we proceed to prove that sole-sourcing is optimal for the buyer.

Proposition 3. *There exists a unique symmetric equilibrium at the investment stage. That equilibrium investment is characterized by x^* satisfying*

$$\int_0^\omega H_2(c|x^*)(\alpha-(2\alpha-1)H(c|x^*))dc = 1. \tag{14}$$

When $1-H(c|x)$ is log-convex in x, the symmetric equilibrium is stable.

The total procurement cost of the buyer equals the sum of suppliers' expected profits at the bidding stage, plus the expected production cost and the investment costs of the suppliers, i.e. $2E\Pi_i(x^*, x^*)+Ec+2x^*$. For the expected production cost, the fraction α of the good will be produced by the low cost supplier and the fraction $1-\alpha$ of the good will be produced by the high cost supplier.

Lemma 2. *When $1-H(c|x)$ is log-convex, the buyer's expected procurement cost as a function of α is*

$$Em(\alpha) = \omega-(2\alpha-1)\int_0^\omega H(c|x^*(\alpha))^2 dc. \tag{15}$$

The derivative of $Em(\alpha)$ with respective to α is given by:

$$\frac{\partial}{\partial\alpha}Em(\alpha) = -2\int_0^\omega H(c|x^*)^2 dc-(2\alpha-1)\int_0^\omega 2H(c|x^*)H_2(c|x^*)\frac{dx^*}{d\alpha}dc. \tag{16}$$

From Eq. (14), we get

$$\frac{dx^*}{d\alpha} = -\frac{\int_0^\omega H_2(c|x^*)(1-2H(c|x^*))dc}{\int_0^\omega (H_{22}(c|x^*)(\alpha-(2\alpha-1)H(c|x^*))-(2\alpha-1)(H_2(c|x^*))^2)dc}. \tag{17}$$

In the next theorem, we establish that sole-sourcing is buyer-optimal when $1-H(c|x)$ is log-convex by showing that $Em(\alpha)$ strictly decreases in α when the equilibrium at the investment stage is symmetric. Therefore, $\alpha=1$ leads to the lowest procurement cost for the buyer.

Theorem 2. *When $1-H(c|x)$ is log-convex in x, it is optimal for the buyer to use sole-sourcing.*

3.2. Log-concave investment

When $1-H(c|x)$ is log-concave, we first show that if $\alpha=1$, the symmetric equilibrium at the investment stage is unstable and the asymmetric pure strategy equilibria are such that one supplier invests positively and the other supplier invests zero.[11] Given such asymmetric outcome of the investment stage, $\alpha=1$ leads to maximal procurement cost ω and optimally splitting the contract lowers the procurement cost.

Proposition 4. *When $1-H(c|x)$ is log-concave in x and $\alpha=1$, the equilibrium investments are given by $(x^{**},0)$ or $(0,x^{**})$, with x^{**} defined by*

$$\int_0^\omega H_2(c|x^{**})dc = 1. \tag{18}$$

[10] When $1-H(c|x)$ is log-convex, there exist no asymmetric equilibria at the investment stage.

[11] Given $\alpha=1$, there exists no pure strategy equilibrium such that two suppliers invest positively but at different levels.

Since asymmetric investment is the equilibrium outcome at the investment stage, a sole sourcing policy leads to a maximal procurement cost of ω. Using a split-award reduces the asymmetry of investment and increases the competitiveness of the bidding stage.

Theorem 3. *When $1-H(c|x)$ is log-concave in x, a split-award is optimal for the buyer.*

To determine the optimal split for the buyer, more details on the investment technology are required. Suppose $H(c|x)$ is three-times differentiable and $H_{222}(c|x)\geq 0$. Then there exists a unique α^* such that the symmetric equilibrium is stable if and only if $\alpha\leq\alpha^*$. At the investment stage, for $\alpha\in(\alpha^*,1]$, the symmetric equilibrium is unstable and there exist asymmetric equilibria such that one firm invests positively and the other firm does not invest. For $\alpha\in\left(\frac{1}{2},\alpha^*\right]$, the symmetric equilibrium exists, is unique and stable.[12,13]

It is straightforward to see that in this case α^* is the unique buyer-optimal split. For $\alpha\in(\alpha^*,1]$, the investment stage outcome is that one firm invests and one firm does not invest, and the buyer's procurement cost is equal to the maximal value ω. For $\alpha\in\left(\frac{1}{2},\alpha^*\right]$, the investment equilibrium is symmetric. As noted in Section 3.1, when investment is symmetric, the buyer's procurement cost decreases with α and is always below ω. Therefore, α^* indeed minimizes the buyer's procurement cost.

We conclude this subsection with a numerical example that satisfies $1-H(c|x)$ log-concave, as well as the other regularity assumptions on $H(c|x)$.

Example 2. Marginal production cost c takes the value of 0 with probability $p(x)$, and takes the value of 1 with probability $1-p(x)$, where $p(x)$ is given by

$$p(x) = \frac{e^{-e^{-x}}-e^{-1}}{1-e^{-1}}. \tag{19}$$

Investment cost is $g(x)=0.25x$. Simulation shows that the optimal split is 77.2% which leads to a cost saving of approximately 9.5% in comparison to sole sourcing.

3.3. Social efficiency

Piccione and Tan (1996) show that in the log-convex case, investment technology exhibits decreasing returns to scale, and efficient investment entails more than one firm. Thus in the log-convex case, we have that more than one firm is socially efficient and buyer-optimal. Moveover, the buyer-optimal policy induces efficient investment, as we see by comparing Eq. (14) to the solution to x^* that maximizes total expected surplus (3) in Piccione and Tan (1996). In contrast, in the log-concave case where investment technology exhibits increasing returns to scale, a single firm investing is efficient from the social optimum perspective but it is not buyer-optimal. There, the socially efficient outcome produces no bidding competition.

Is log-concavity of $1-H(c|x)$ empirically relevant? In many R&D situations, investment by several firms comes at a substantial cost.

[12] A detailed proof of the statement can be found in the Supplementary Technical Appendix.

[13] Note that $\frac{H_2(x)}{\alpha-(2\alpha-1)H(x)}$ monotonically increases in x for $\alpha\sim 1$ and decreases in x for $\alpha\sim\frac{1}{2}$. If we further assume that there exists a $\tilde{\alpha}$ such that $\frac{H_2(x)}{\alpha-(2\alpha-1)H(x)}$ monotonically increases in x for $\alpha>\tilde{\alpha}$ and decreases in x for $\alpha<\tilde{\alpha}$, we can ensure that the investment stage equilibria are either asymmetric with one firm investing zero or symmetric. In the former case, any asymmetric equilibria where both firms invest positively are destabilized by an equilibrium where one firm invests zero. In the latter case, any asymmetric equilibria are destabilized by the symmetric equilibrium.

Innovations by two firms are not combined in the final product, and inventions and discoveries may be duplicated. These considerations—redundancy and lack of integration—support the log-concavity assumption. A diseconomy of scale at the firm level, like the cost of managing a larger team or the problem that firms may have difficulty in investing in several competing R&D approaches, would support log-convexity. Overall, both log-convexity and log-concavity appear plausible, depending on the detailed description of the environment.

4. Extensions and discussions

In this section, we first show that the deterministic model in Section 2 can be treated as a special case of the general stochastic model in Section 3 and thus the two models can be unified under the umbrella of log-concavity. Then we examine the robustness of the optimality of split-award in several directions.

4.1. Comparison of the stochastic and deterministic models

Note that the cost of investment in the deterministic case can be normalized as $x=\hat{g}(c)$. Use t to denote cost reduction below ω. Then we have $\omega - t=\hat{g}^{-1}(x)$. The investment technology with deterministic outcome in Section 2 can then be written as:

$$H(t|x)=\begin{cases}1 & \text{if } t \geq \omega - \hat{g}^{-1}(x),\\ 0 & \text{if } t < \omega - \hat{g}^{-1}(x).\end{cases} \tag{20}$$

For given x, $H(t|x)$ is a distribution function with all the mass at $t=\omega-\hat{g}^{-1}(x)$. Distribution (20) can be approximated by a distribution function that exhibits log-concavity.[14] Therefore, we can unify the results in the deterministic model and stochastic model under the assumption of log-concavity of the investment technology.

4.2. Reserve price

Suppose when the buyer chooses the split α, she determines a reserve price r and announces it to the suppliers. At the bidding stage, if each supplier places a bid below r, the allocation rule remains the same as before. If both suppliers place a bid above r, no contract is awarded. If one supplier places a bid above r and the other with a bid below r, the supplier with low bid is awarded the entire contract at a price equal to r.

In the deterministic case, sole-sourcing with a reserve price is efficient and buyer-optimal. Suppose the reserve price r is such that $r-c^*-g(c^*)=0$ where c^* is the solution to $-1=g'(c)$. At the investment stage, only one firm investing is sustained, which is efficient. The investing firm obtains the entire contract at a price equal to r and the buyer extracts the entire surplus by choosing a reserve price such that the investing firm receives zero expected payoff. In this case, a reserve price is a substitute for a second competitor. Nevertheless, the reserve is risky—suppose the buyer has slight uncertainty about $g(\cdot)$ and the reserve is set just below $c^*+g(c^*)$, no one will be willing to bid, even if the value of the contract to the buyer is enormous. In contrast, using a split-award contract "fails gracefully" when errors are made in setting values.

In the stochastic case where investment only determines the distribution of marginal costs and the realized marginal costs are private information of the suppliers, in the log-concave case, from ex ante point of view, using sole-sourcing plus a reserve price can elicit efficient investment and drive the ex ante expected payoff of the suppliers down to zero. However, for any reserve price $r<\omega$, there is a strict positive probability that the contract is not awarded. For moderate values of the contract, reserve prices may dominate split-awards. Nevertheless, if the loss from not implementing the contract is large, setting a reserve price below ω is not buyer-optimal and split-award is still preferred by the buyer to sole-sourcing.

4.3. Multiple suppliers

The presence of more than two suppliers demands split-award for the same reason as the case of two suppliers: split-award encourages at least one additional supplier to constrain the pricing of the lowest cost supplier. Generally split-award creates a problem of multiple equilibria, because weakly dominant bids no longer exist. This is a familiar problem from search auctions (see, for example, Edelman and Ostrovsky (2007) and Edelman et al. (2007)), where typically the envy-free equilibrium is selected.

We consider the case of three suppliers, when suppliers are indifferent to tying with the next lower cost firm, and the lowest cost firm indifferent with the middle firm at the bidding stage. As in example 1, we assume quadratic costs. In this case, it is buyer-optimal to split the contract among the suppliers by awarding a positive share to each one. The optimal splits in this numerical example are roughly 64.5%, 28%, and 7.5%, independent of δ and ω. Thus, a split-award among two firms is not generally optimal in the multiple firm case. Moreover, the share of the largest firm falls with three firms. Somewhat surprisingly, the optimal share of the second largest firm rises with three firms, from 25% to 28%. We do not know if this is a general property and further research will be needed to confront the multiple equilibrium problem inherent with many suppliers.

4.4. First price auctions

In our main analysis, we have been using second price auctions for simplicity. Sealed-bid first price auctions where a supplier is paid his own bid as long as that bid is below ω pose technical difficulties as they involve solving simultaneous equations with asymmetric inverse bidding functions. In a setup similar to Example 2 where the marginal cost takes binary values, we find that the outcome of the bidding stage under the first price auction is the same to that of the second price auction. If the outcomes of the bidding stage are identical, the suppliers' investment decisions at the investment stage and the buyer's optimal choice of α must also be identical under the two auction formats in this particular case.

4.5. Equilibria in mixed strategies

Proposition 2 and Proposition 4 involve asymmetric pure strategy equilibria where one firm invests more than the other firm. Since this may involve a coordination issue, one naturally wonders what happens with mixed strategy equilibria. Consider the deterministic case and suppose after the suppliers make simultaneous investments, information about marginal costs is revealed and the suppliers then submit simultaneous bids. At the bidding stage, the suppliers still follow the equilibrium bidding strategies as prescribed in Proposition 1.

In such a case, the unique symmetric mixed strategy equilibrium of the investment stage is that both suppliers randomize on $[c_L^*, c_H^*]$ according to a distribution function $F(c)=\frac{\alpha+g'(c)}{2\alpha-1}$, where c_L^* and c_H^* are defined in Eq. (6). In this mixed strategy equilibrium, both firms' payoffs are constant and equal to $\pi_H(c_H^*)=(1-\alpha)(\omega-c_H^*)-g(c_H^*)$. For the suppliers, the outcomes of Proposition 2 obviously pareto-dominate the outcomes of this mixed strategy equilibrium. Furthermore, the mixed strategy equilibrium is unstable because a slight perturbation of a player's beliefs about his opponent's strategy upsets the equilibrium.

[14] The proof is done by constructing a distribution function that approximates the outcome of Eq. (20) and then showing that such a function is log-concave. See the Supplementary Technical Appendix for the details.

4.6. Optimal procurement mechanism

As discussed in Section 4.2, with an appropriate reserve price, sole-sourcing is buyer-optimal. However, the informational requirement of this solution is substantial. To set the right reserve, the buyer needs to know the distributions of the marginal costs, the solution is subject to subgame perfection, and the payment will sometimes not cover costs. The latter can be handled by having the suppliers post a bond in advance, so that the payment of the reserve plus repayment of the bond is always enough to cover ω, the maximal cost. This is in fact buyer-optimal in the class of mechanism design because it minimizes cost and gives all the proceeds to the buyer. Like many mechanism design solutions, we don't find it plausible and think a simpler institution, like a fixed split-share, is more reasonable, because the mechanism is robust and easy to implement.

5. Concluding remarks

The basic insight of the paper is that the optimality of dual-sourcing depends on the socially efficient number of firms at the investment stage. When that number is greater than one, sole-sourcing is typically buyer-optimal. When that number is one, dual-sourcing lowers buyer costs. The reason for our result is that auctions induce efficient investment, and efficient investment is not always in the buyer's interest. When more than one firm investing is efficient because there is a diseconomy of scale in investment, the auctions induce these firms to invest and the buyer benefits from robust competition by suppliers even in a winner-take-all scenario. In this case, the winner-take-all scenario serves the buyer well. In contrast, when one firm investing is efficient, a buyer with a winner-take-all, sole-source auction faces one strong and one weak supplier. Inducing a little artificial competition, via the split-award, is advantageous to the buyer. Our main contribution is to develop this intuition and show that it is quite robust.

In our model, the sequence of the game depends on the commitment power of the buyer. If the buyer lacks commitment power or the sequence of the game is reversed such that investment occurs before the buyer announces the procurement policy, the equilibrium/buyer-optimal procurement policy is always sole-sourcing. In that case, one supplier invests and one supplier does not at the investment stage. The resulting procurement cost for the buyer is maximized at ω which is the highest possible contract cost in our setting.

In the paper, we have focused on the single fixed split because it lets us readily demonstrate the optimality of some split, in contrast to much of the literature, and highlight the use of split awards in creating effective competition through investment. More general splitting mechanisms may improve on the outcomes contained in the present study and we leave that for future research.

Appendix A

Proof of Proposition 1. By bidding b_i, supplier i with cost c_i is the low bidder and receives $\alpha(y-c_i)$ if bidder j's bid $y \geq b_i$. Supplier i is the high bidder and receives $(1-\alpha)(\omega-c_i)$ if $y<b_i$.

Consider a deviating bid $z>b_i$. If $y>z>b_i$, supplier i is still the low bidder and his payoff does not improve from the deviation. If $z>b_i>y$, supplier i is still the high bidder and there is no improvement from the deviation either. If $z>y>b_i$, supplier i becomes the high bidder instead and receives $(1-\alpha)(\omega-c_i)$. This is equal to $\alpha(b_i-c_i)$, which is smaller than $\alpha(y-c_i)$, supplier i's payment if he had bid b_i and become the low bidder. Therefore, an upward deviation never improves bidder i's payoff and sometimes decreases his payoff strictly.

Similarly, any downward deviation is not profitable for supplier i either.

Proof of Proposition 2. First, (c_L^*, c_H^*) as defined in Eq. (6) are unique interior solutions to Eqs. (4) and (5) and are mutually best responses.

Second, for the one to be the high cost supplier, choosing c_H^* guarantees a positive payoff while choosing $c_H=\omega$ leads to a payoff of 0. On the other hand, choosing $c_H=0$ never pays either as $g'(0)<-1$ and it is not worthwhile to reduce cost to the level of 0. Therefore, choosing c_H^* is strictly better than choosing one of the corners for a supplier that turns out to be the high cost one. Similar argument applies if the supplier turns out to be the low cost one.

Proof of Theorem 1. The derivative of the buyer's procurement cost (7) with respect to α is:

$$\frac{dm(\alpha)}{d\alpha} = -2\omega + 2c_H + 2\alpha\frac{dc_H}{d\alpha}. \tag{21}$$

When $\alpha=1$, $c_H=\omega$. From Eq. (6), we have

$$\frac{dc_H}{d\alpha} = \frac{1}{g''(c_H)}. \tag{22}$$

Hence,

$$\left.\frac{dm(\alpha)}{d\alpha}\right|_{\alpha=1} = \frac{2}{g''(c_H)} > 0. \tag{23}$$

Therefore, at $\alpha=1$, a marginal reduction of the share α lowers the buyer's procurement cost.

Proof of Lemma 1. Consider supplier i with marginal cost c_i. Supplier i is the winner if j's marginal cost is above c_i, which occurs with probability $1-H(c_i|x_j)$, where $H(\cdot|x_j)$ is supplier j's distribution of marginal cost given his investment x_j. Supplier i is the loser if j's marginal cost is below c_i. That occurs with probability $H(c_i|x_j)$. Therefore, given his own marginal cost c_i, supplier i's expected payoff is:

$$\begin{aligned}
\pi(c_i|x_j) &= H(c_i|x_j)(1-\alpha)(\omega-c_i) + (1-H(c_i|x_j))\alpha(E[b(C_j)|C_j > c_i]-c_i) \\
&= H(c_i|x_j)(1-\alpha)(\omega-c_i) \\
&\quad +(1-H(c_i|x_j))\alpha\left(\frac{1}{1-H(c_i|x_j)}\int_{c_i}^{\omega} b_j(t)h(t|x_j)dt - c_i\right) \\
&= H(c_i|x_j)(1-\alpha)(\omega-c_i) + \alpha\int_{c_i}^{\omega}\left(\frac{1-\alpha}{\alpha}(\omega-t)+t\right)dH(t|x_j) - (1-H(c_i|x_j))\alpha c_i \\
&= \alpha(\omega-c_i) - (2\alpha-1)\int_{c_i}^{\omega} H(c_j|x_j)dc_j.
\end{aligned}$$

At the investment stage, the two suppliers simultaneously choose their investment levels which affect the payoffs at the bidding stage. By investing x_i, the distribution of marginal cost of supplier i is $H(c|x_i)$. Therefore, his expected payoff at the investment stage is

$$\begin{aligned}
\Pi_i(x_i, x_j) &= E[\pi(c_i|x_j) - x_i|x_i] \\
&= \int_0^{\omega}(\alpha(\omega-c_i) - (2\alpha-1)\int_{c_i}^{\omega} H(c_j|x_j)dc_j)dH(c_i|x_i) - x_i \\
&= \int_0^{\omega} H(c_i|x_i)(\alpha - (2\alpha-1)H(c_i|x_j))dc_i - x_i.
\end{aligned}$$

Proof of Proposition 3. First, since $H_{22}(c_i|x_i) \leq 0$ by assumption, $\alpha-(2\alpha-1)H(c_i|x_j)>0$ since $\alpha > \frac{1}{2}$ and $H(\cdot) \leq 1$, and SOC (13) is negative for every x_i, $\Pi_i(x_i, x_j)$ is globally concave in x_i and the solution determined by the first order condition is globally optimal. Denote the solution as (x_i^*, x_j^*).

Second, since $sign\frac{dx_i^*}{dx_j^*}=sign\frac{\partial^2\Pi_i}{\partial x_i\partial x_j}$ and

$$\frac{\partial^2\Pi_i}{\partial x_i\partial x_j}=-(2\alpha-1)\int_0^\omega H_2(c_i|x_i)H_2(c_i|x_j)dc_i<0, \tag{24}$$

a symmetric equilibrium is uniquely determined by (14). Set Eq. (12) equal to zero and denote $x_i(x_j)$ as its solution. Note that

$$\begin{aligned}\lim_{x_i\to\infty}\Pi_i(x_i(x_j)) &= \lim_{x_i\to\infty}\int_0^\omega(\alpha(\omega-c_i)-(2\alpha-1)\int_{c_i}^\omega H(c_j|x_j)dc_j)dH(c_i|x_i)-x_i\\ &\le \lim_{x_i\to\infty}\int_0^\omega\alpha(\omega-c_i)dH(c_i|x_i)-x_i\\ &\le \lim_{x_i\to\infty}\alpha\omega-x_i<0.\end{aligned}$$

Hence, $x_i(0)<\infty$. Further, given that supplier j invests $x_j=x^*$, we have

$$\begin{aligned}\frac{\partial\Pi_i}{\partial x_i}\Big|_{(0,x^*)} &= \int_0^\omega H_2(c_i|0)(\alpha-(2\alpha-1)H(c_i|x^*))dc_i-1\\ &= \int_0^\omega H_2(c_i|0)(\alpha-(2\alpha-1)H(c_i|x^*))dc_i\\ &\quad-\int_0^\omega H_2(c_i|x^*)(\alpha-(2\alpha-1)H(c_i|x^*))dc_i\\ &= \int_0^\omega(H_2(c_i|0)-H_2(c_i|x^*))(\alpha-(2\alpha-1)H(c_i|x^*))dc_i>0.\end{aligned}$$

where the inequality holds because $H_{22}(c_i|x^*)\le 0$ which leads to $H_2(c_i|0)>H_2(c_i|x^*)$. Hence, $\frac{\partial\Pi_i}{\partial x_i}\Big|_{(0,x^*)}>0$. Thus, the symmetric equilibrium indeed exists and is unique.

From Eq. (12), we have:

$$\frac{dx_i}{dx_j}=\frac{\int_0^\omega H_2(c_i|x_i)(2\alpha-1)H_2(c_i|x_j)dc_i}{\int_0^\omega H_{22}(c_i|x_i)(\alpha-(2\alpha-1)H(c_i|x_j))dc_i.}$$

Evaluate this at $x_i=x_j=x^*$, we get

$$\frac{dx_i}{dx_j}\Big|_{x_i=x_j=x^*}=\frac{\int_0^\omega H_2(c_i|x^*)^2(2\alpha-1)dc_i}{\int_0^\omega H_{22}(c_i|x^*)(\alpha-(2\alpha-1)H(c_i|x^*))dc_i}.$$

By log-convexity of $1-H(c|x)$, we have $H_2(c|x)^2<-H_{22}(c|x)(1-H(c|x))$. Since

$$-H_{22}(c|x)(2\alpha-1)(1-H(c|x))\le -H_{22}(c|x)(\alpha-(2\alpha-1)H(c|x)),$$

for every c_i, we have

$$H_2(c_i|x^*)^2(2\alpha-1)\le -H_{22}(c|x^*)(\alpha-(2\alpha-1)H(c|x^*)).$$

Therefore, $\left|\frac{dx_i}{dx_j}\Big|_{x_i=x_j=x^*}\right|\le 1$. Hence, the symmetric equilibrium is stable.

To complete the proof, we show that when $1-H(c|x)$ is log-convex, there do not exist asymmetric equilibria. Suppose there is an asymmetric equilibrium with $x_i^*<x_j^*$. x_j^* must be a best response to x_i^* and satisfy:

$$\int_0^\omega H_2(c|x_j^*)(\alpha-(2\alpha-1)H(c|x_i^*))dc=1$$

Nevertheless, given $x_j=x_j^*$, it is beneficial for player i to unilaterally increase his investment above x_i^* because

$$\begin{aligned}\frac{\partial\Pi_i}{\partial x_i}\Big|_{(x_i^*,x_j^*)} &= \int_0^\omega H_2(c|x_i^*)(\alpha-(2\alpha-1)H(c_i|x_j^*))dc-1\\ &= \int_0^\omega H_2(c|x_i^*)(\alpha-(2\alpha-1)H(c|x_j^*))dc\\ &\quad-\int_0^\omega H_2(c|x_j^*)(\alpha-(2\alpha-1)H(c_i|x_i^*))dc\\ &>0.\end{aligned}$$

The inequality holds because $r(c,x)$ decreases in x, which means that $\frac{H_2(c|x)}{\alpha-(2\alpha-1)H(c|x)}$ decreases in x. Thus, when $1-H(c|x)$ is log-convex, there do not exist asymmetric equilibria at the investment stage.

Proof of Lemma 2. From the buyer's perspective, the fraction α of the good will be produced by the low cost supplier and the fraction $1-\alpha$ of the good will be produced by the high cost supplier. The expected production cost is:

$$\begin{aligned}Ec &= \alpha\int_0^\omega c2(1-H(c|x^*))h(c|x^*)dc+(1-\alpha)\int_0^\omega c2H(c|x^*)h(c|x^*)dc\\ &= \alpha\int_0^\omega(1-H(c|x^*))^2dc+(1-\alpha)\int_0^\omega(1-H(c|x^*)^2)dc.\end{aligned}$$

The total procurement cost of the buyer equals the sum of suppliers' expected profits at the bidding stage, plus the expected production cost and the investment costs of the suppliers.

$$\begin{aligned}Em(\alpha) &= 2E\Pi_i(x^*,x^*)+Ec+2x^*\\ &= 2\int_0^\omega H(c|x^*)(\alpha-(2\alpha-1)H(c|x^*))dc\\ &\quad+\alpha\int_0^\omega(1-H(c|x^*))^2dc+(1-\alpha)\int_0^\omega(1-H(c|x^*)^2)dc\\ &= \omega-(2\alpha-1)\int_0^\omega H(c|x^*)^2dc.\end{aligned}$$

Proof of Theorem 2. The aim is to show that $Em(\alpha)$ monotonically decreases in α for $\alpha\in(\frac{1}{2},1]$. The proof is done in two steps. In Step 1, we show that $Em(\alpha)$ decreases at $\alpha=1$. In Step 2, we show that for $\alpha\in(\frac{1}{2},1)$, $Em(\alpha)$ must be monotonically decreasing.

Step 1. $Em(\alpha)$ decreases at $\alpha=1$, i.e. $\frac{\partial Em(\alpha)}{\partial\alpha}|_{\alpha=1}<0$. The proof is done by contradiction. Suppose $\frac{\partial}{\partial\alpha}Em(\alpha)|_{\alpha=1}\ge 0$ holds instead. This requires:

$$-2\left(\int_0^\omega H(c|x^*)^2dc+\int_0^\omega H(c|x^*)H_2(c|x^*)\frac{dx^*}{d\alpha}|_{\alpha=1}dc\right)\ge 0, \tag{25}$$

which is equivalent to

$$\int_0^\omega H(c|x^*)^2dc+\int_0^\omega H(c|x^*)H_2(c|x^*)\frac{dx^*}{d\alpha}|_{\alpha=1}dc\le 0. \tag{26}$$

From Eq. (17), we have:

$$\frac{dx^*}{d\alpha}\Big|_{\alpha=1}=-\frac{\int_0^\omega H_2(c|x^*)(1-2H(c|x^*))dc}{\int_0^\omega(H_{22}(c|x^*)(1-H(c|x^*))-(H_2(c|x^*))^2)dc}. \tag{27}$$

Plugging Eq. (27) into Eq. (26) and rearranging, and noting that the denominator of Eq. (27) is negative, for $\frac{\partial}{\partial\alpha}Em(\alpha)|_{\alpha=1}>0$, we need

$$\begin{aligned}&\int_0^\omega H(c|x^*)^2dc\int_0^\omega(H_{22}(c|x^*)(1-H(c|x^*))-(H_2(c|x^*))^2)dc\\ &\quad-\int_0^\omega H(c|x^*)H_2(c|x^*)dc\int_0^\omega H_2(c|x^*)(1-2H(c|x^*))dc\le 0.\end{aligned} \tag{28}$$

Rearranging Eq. (28) delivers

$$\int_0^{\omega} H(c|x^*)^2 dc \int_0^{\omega} H_{22}(c|x^*)(1-H(c|x^*))dc - \int_0^{\omega} H(c|x^*)H_2(c|x^*)dc \int_0^{\omega} H_2(c|x^*)(1-H(c|x^*))dc - \left\{ \int_0^{\omega} H(c|x^*)^2 dc \int_0^{\omega} H_2(c|x^*)^2 dc - \left(\int_0^{\omega} H(c|x^*)H_2(c|x^*)dc \right)^2 \right\} \le 0. \tag{29}$$

The first term on LHS is negative since $H_{22}(c|x^*)<0$ by assumption. The second term is positive since $H_2(c|x)>0$ by assumption. Further,

$$\int_0^{\omega} H(c|x^*)^2 dc \int_0^{\omega} H_2(c|x^*)^2 dc \ge \left(\int_0^{\omega} H(c|x^*)H_2(c|x^*)dc \right)^2$$

by Cauchy Schwarz inequality, and hence the third term in the curly bracket on LHS is nonnegative. Therefore, Eq. (29) never holds, which implies that $\frac{\partial}{\partial\alpha}Em(\alpha)|_{\alpha=1}<0$ must hold.

Step 2. $Em(\alpha)$ must be monotonically decreasing in α for $\alpha\in(\frac{1}{2},1)$. First of all, from Eq. (15), we see that

$$\lim_{\alpha\to\frac{1}{2}} \frac{\partial}{\partial\alpha}Em(\alpha) = -2\int_0^{\omega} H(c|x^*)^2 dc<0,$$

which implies that $Em(\alpha)$ decreases as α marginally moves away from $\frac{1}{2}$. Suppose $Em(\alpha)$ is nonmonotonic in α on $(\frac{1}{2},1)$ and $\frac{\partial}{\partial\alpha}Em(\alpha)$ changes signs at some $\tilde{\alpha}$s. There are at least two such $\tilde{\alpha}$s and each one can be either a local maximum or minimum of $Em(\alpha)$. For any of such $\tilde{\alpha}$, one must have $\frac{\partial}{\partial\alpha}Em(\alpha)|_{\alpha=\tilde{\alpha}}=0$. Denote the equilibrium investment for $\tilde{\alpha}$ as $x^*(\tilde{\alpha})$. From Eq. (16), it must hold that $\frac{dx^*}{d\alpha}|_{\alpha=\tilde{\alpha}}<0$.

Recall that

$$\frac{dx^*}{d\alpha} = -\frac{\int_0^{\omega} H_2(c|x^*)(1-2H(c|x^*))dc}{\int_0^{\omega} \left(H_{22}(c|x^*)(\alpha-(2\alpha-1)H(c|x^*))-(2\alpha-1)(H_2(c|x^*))^2\right)dc}.$$

Since the denominator on the RHS of the above equation is negative, $\frac{dx^*}{d\alpha}|_{\alpha=\tilde{\alpha}}<0$ implies $\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))(1-2H(c|x^*(\tilde{\alpha})))dc<0$.

From Eq. (14), we have:

$$\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))(\tilde{\alpha}-(2\tilde{\alpha}-1)H(c|x^*(\tilde{\alpha})))dc = \int_0^{\omega} H_2(c|x^*(1))(1-H(c|x^*(1)))dc.$$

This leads to

$$\tilde{\alpha} = \frac{\int_0^{\omega} H_2(c|x^*(1))(1-H(c|x^*(1)))dc - \int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))H(c|x^*(\tilde{\alpha}))dc}{\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))(1-2H(c|x^*(\tilde{\alpha})))dc}.$$

Thus,

$$2\tilde{\alpha}-1 = \frac{2\int_0^{\omega} H_2(c|x^*(1))(1-H(c|x^*(1)))dc - \int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))dc}{\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))(1-2H(c|x^*(\tilde{\alpha})))dc},$$

from which we get

$$\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))(1-2H(c|x^*(\tilde{\alpha})))dc = \frac{1}{2\tilde{\alpha}-1}\left(2\int_0^{\omega} H_2(c|x^*(1))(1-H(c|x^*(1)))dc - \int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))dc\right) = \frac{1}{2\tilde{\alpha}-1}\left(2-\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))dc\right).$$

$\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))(1-2H(c|x^*(\tilde{\alpha})))dc<0$ requires that $\int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))dc>2$. From Eq. (14), we have $\lim_{\alpha\to\frac{1}{2}}\int_0^{\omega} H_2(c|x^*(\alpha))dc=2$. Therefore,

$$\lim_{\alpha\to\frac{1}{2}} \int_0^{\omega} H_2(c|x^*(\alpha))dc < \int_0^{\omega} H_2(c|x^*(\tilde{\alpha}))dc. \tag{30}$$

Since $H_2(c|x)$ is a decreasing function in x, Eq. (30) implies that $\lim_{\alpha\to\frac{1}{2}}x^*(\alpha)>x^*(\tilde{\alpha})$. Since $Em(\alpha)$ is monotonically decreasing in x, it implies $Em(\tilde{\alpha})>\lim_{\alpha\to\frac{1}{2}}Em(\alpha)$. This leads to a contradiction since from Eq. (15), we have

$$\lim_{\alpha\to\frac{1}{2}} Em(\alpha) = \omega > \omega-(2\tilde{\alpha}-1)\int_0^{\omega} H(c|x^*(\tilde{\alpha}))^2 dc = Em(\tilde{\alpha}).$$

In summary, since $Em(\alpha)$ decreases at $\alpha=1$ and is monotonic on $(\frac{1}{2},1)$, $Em(\alpha)$ must be strictly decreasing on $(\frac{1}{2},1]$. Therefore, $Em(\alpha)$ takes its minimum value at $\alpha=1$ and sole-sourcing is optimal for the buyer.

Proof of Proposition 4. The proof is done in three steps. In Step 1, we show that the proposed candidate is indeed an equilibrium at the investment stage. In Step 2, we show that there exists no asymmetric equilibrium in which both firms invest positively. In Step 3, we show that the symmetric equilibrium is unstable.

Step 1. When $\alpha=1$, $(x^{**},0)$ and $(0,x^{**})$ are equilibria of the investment stage. When $\alpha=1$, supplier i's payoff at the investment stage is:

$$\Pi_i(x_i,x_j) = \int_0^{\omega} H(c_i|x_i)(1-H(c_i|x_j))dc_i - x_i. \tag{31}$$

Suppose $x_j=0$, the best response of supplier i is given by $x_i=x^{**}$. Suppose supplier j has invested x^{**}, for $x_i\le x^{**}$, we have

$$\left.\frac{\partial \Pi_i}{\partial x_i}\right|_{x_j=x^{**}} = \int_0^{\omega} H_2(c_i|x_i)\cdot(1-H(c_i|x^{**}))dc_i - 1 = \int_0^{\omega} (H_2(c_i|x_i)\cdot(1-H(c_i|x^{**}))-H_2(c_i|x^{**}))dc_i \le \int_0^{\omega} H_2(c_i|x^{**})\cdot(1-H(c_i|x_i)-H_2(c_i|x^{**}))dc_i = -\int_0^{\omega} H_2(c_i|x^{**})\cdot H(c_i|x_i)dc_i \le 0,$$

where the inequality of the third line holds because $1-H(c|x)$ is log-concave and $r(c_i,x_i)$ is an increasing function in x_i. $\left.\frac{\partial \Pi_i}{\partial x_i}\right|_{x_j=x^{**}}\le 0$ implies that 0 is indeed a best response to x^{**}. Therefore, $(0,x^{**})$ and $(x^{**},0)$ are equilibria of the investment stage for $\alpha=1$.

Step 2. When $\alpha=1$, there exists no asymmetric equilibria where both firms invest positively. Suppose there exist an asymmetric equilibrium such that $0<\hat{x}_i<\hat{x}_j$. Then $(\hat{x}_i,\hat{x}_j)$ must satisfy:

$$\int_0^{\omega} H_2(c|\hat{x}_j)(1-H(c|\hat{x}_i))dc = 1. \tag{32}$$

Suppose supplier j invests $x_j=\hat{x}_j$, for supplier i, we have

$$\left.\frac{\partial \Pi_i}{\partial x_i}\right|_{x_i=\hat{x}_i} = \int_0^{\omega} H_2(c|\hat{x}_i)\cdot(1-H(c_i|\hat{x}_j))dc - 1 = \int_0^{\omega} H_2(c|\hat{x}_i)\cdot(1-H(c_i|\hat{x}_j))dc - \int_0^{\omega} H_2(c|\hat{x}_j)(1-H(c|\hat{x}_i))dc < 0,$$

where the inequality of the third line holds because $1-H(c|x)$ is log-concave and $r(c_i,x_i)$ is an increasing function in x_i. Therefore, there exists no asymmetric equilibrium where both firms invest positively.

Step 3. The symmetric equilibrium is unstable. Suppose there exists an equilibria $(\hat{x}_i, \hat{x}_j)$ where both suppliers invest positively. For $\alpha = 1$, $(\hat{x}_i, \hat{x}_j)$ must satisfy

$$\int_0^{\omega} H_2(c_i|\hat{x}_i)(1 - H(c_i|\hat{x}_j))dc_i = 1.$$

For a symmetric equilibrium, we have

$$\frac{d\hat{x}_i}{d\hat{x}_j}\bigg|_{x_i = x_j = \hat{x}} = \frac{\int_0^{\omega} H_2(c|\hat{x}_i) H_2(c|\hat{x}_j)dc}{\int_0^{\omega} H_{22}(c|\hat{x}_i)(1 - H(c|\hat{x}_j))dc}\bigg|_{x_i = x_j = \hat{x}}$$
$$= \frac{\int_0^{\omega} (H_2(c|\hat{x}))^2 dc}{\int_0^{\omega} H_{22}(c|\hat{x})(1 - H(c|\hat{x}))dc}.$$

Log-concavity of $1 - H(c|x)$ implies

$$H_{22}(c|x)(1 - H(c|x)) \geq -(H_2(c|x))^2.$$

Hence, $\frac{d\hat{x}_i}{d\hat{x}_j}\Big|_{x_i = x_j = \hat{x}} < -1$. Therefore, at $\alpha = 1$, the symmetric equilibrium is unstable if $1 - H(c|x)$ is log-concave.

Proof of Theorem 3. From Proposition 4, the equilibrium at the investment stage for $\alpha = 1$ is that one supplier invests x^{**} while the other supplier does not invest. Given this outcome, one supplier's marginal cost is ω, which leads to a procurement price of $Em(1) = \omega$ at the bidding stage.

In the following, we argue that there exists an $\alpha \in \left(\frac{1}{2}, 1\right)$ such that at the investment stage, there exists an equilibrium where both suppliers invest positively. If both suppliers invest positively, the resulting procurement cost for the buyer must be below ω. Recall that a supplier's expected payoff at the investment stage is:

$$\Pi_i(x_i, x_j) = \int_0^{\omega} H(c_i|x_i)(\alpha - (2\alpha - 1)H(c_i|x_j))dc_i - x_i.$$

Evaluating $\frac{dx_i}{dx_j}$ at $x_i = x_j$ gives:

$$\frac{dx_1}{dx_2}\bigg|_{x_1 = x_2 = x^*} = \frac{\int_0^{\omega} H_2(c_i|x_i)(2\alpha - 1)H_2(c_i|x_j)dc_i}{\int_0^{\omega} H_{22}(c_i|x_i)(\alpha - (2\alpha - 1)H(c_i|x_j))dc_i}\bigg|_{x_i = x_j = x^*}$$
$$= \frac{\int_0^{\omega} H_2(c_i|x^*)^2 (2\alpha - 1)dc_i}{\int_0^{\omega} H_{22}(c_i|x^*)(\alpha - (2\alpha - 1)H(c_i|x^*))dc_i}. \quad (33)$$

When $\alpha = 1$, it has been proven in Proposition 4 that $\frac{dx_1}{dx_2}\Big|_{x_1 = x_2 = x^*} < -1$, which implies that the symmetric equilibrium is unstable.

Further note that $lim_{\alpha \to \frac{1}{2}} \frac{dx_1}{dx_2}\Big|_{x_1 = x_2 = x^*} = 0 \in [-1, 1]$ and in that case, the symmetric equilibrium exists and is stable. Therefore, there must exist a smallest $\hat{\alpha}$ such that for $\alpha \in \left(\frac{1}{2}, \hat{\alpha}\right)$, the symmetric equilibrium exists and is stable. (For $\alpha \to \frac{1}{2}$, $\frac{H_2(x)}{\alpha - (2\alpha - 1)H(x)}$ is a monotonically decreasing function and any asymmetric equilibria are destabilized by the symmetric equilibrium.) In the symmetric equilibrium, both suppliers invest positively and hence the procurement cost must be below ω as long as $\alpha > \frac{1}{2}$.

Appendix B. Supplementary Technical Appendix

Supplementary Technical Appendix to this article can be found online at doi:10.1016/j.jpubeco.2011.10.001.

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