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Discrete Equilibrium Price Dispersion

John A. Carlson
Purdue University

R. Preston McAfee
University of Western Ontario

An explicit solution of an equilibrium model with price-setting firms and searching customers makes possible a number of comparative-statics predictions about how cost differences among firms, search costs of customers, and taxes will affect the mean and variance of the distribution of market prices. Another implication of the model is that a firm's demand depends on the difference between its price and the average price in the market.

I. Introduction

In his seminal article on the economics of information, Stigler (1961) wrote that price "dispersion is ubiquitous even for homogeneous goods." He cited two examples. One was a model of Chevrolet automobile in Chicago in 1959. After an average amount of "haggling," the mean price among 27 dealers was $2,436 with a standard deviation of $42. The other commodity was anthracite coal, delivered. The average asking price of 14 dealers in Washington, D.C., April 1953, was $16.90 with a standard deviation of $1.15. Stigler suggested a number of hypotheses about what would influence such differences.

Almost 2 decades later, when Pratt, Wise, and Zeckhauser (1979), hereafter PWZ, report on price quotes in a telephone sample of 39

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items, they refer (p. 205) to the "surprisingly large difference among prices that we observed . . . ." Why they consider their results to be surprising is not immediately evident. The dispersions do not appear unusual relative to data from other places or other times. Of their 39 products, only one has a standard deviation in excess of what Stigler reported for automobile prices. In addition, 12 of the products have a coefficient of variation (ratio of the standard deviation to the mean price) of less than 10 percent, comparable to the 6.8 percent coefficient (1.15/16.90) in Stigler's price data for coal. All of these 12 are well-defined brands or are likely to have very few perceived quality differences. Not very many more of the products in their sample fit those criteria. Most have an important service component or may be perceived as nonhomogeneous (e.g., diamond appraisals, repairing clarinets). For the clearly standardized products the coefficients of variation are similar to data from New Orleans reported by Carlson and Pescatrace (1980).

Furthermore, PWZ do not present a null hypothesis that must be rejected because the observed price differences are too large relative to some critical level, unless they have in mind models that do not produce any dispersion. Certainly, the accumulated evidence, including their own, is of extensive price dispersion. To make progress analyzing this phenomenon calls for more careful modeling of what can influence the extent of such dispersion.

Telser (1978, chap. 9) claims that the portion of price dispersion attributable to ignorance of prices and costs of search is small for the products he examined: retail gasoline, canned and frozen juices, and market baskets of grocery products. He argues that most of the dispersion reflects equalizing price differences attributable to heterogeneous characteristics being offered to the buyers. This may be correct, although PWZ report that none of the sellers they queried made any claims about accompanying services. The empirical importance of imperfect information in different product markets is surely still an open question.

Much of the recent theoretical work in this area was stimulated by Rothschild's (1973) excellent survey for which Stigler's article, "directly or indirectly, inspired most of the work here surveyed." While he has occasional warnings about models being distant from the real world, what really attracted professional attention was Rothschild's criticism that Stigler's theory considered only one side of the market. The price configuration, assumed known by searching buyers, may not be consistent with the array of prices that would be set by firms who know how buyers search.

Rothschild suggested that we should develop equilibrium models of price dispersion in which both buyers and sellers make optimal use of
the imperfect information they do possess. As a result of subsequent efforts, there now are a number of logically consistent models of price dispersion. Various combinations of assumptions are known to result in no equilibrium, a single-price equilibrium, or an equilibrium with a nondegenerate distribution of prices. Work in this area includes Arrow and Rothschild (1973), Axell (1977), Butters (1977), Salop and Stiglitz (1977), Burdett and Judd (1979), Reinganum (1979), Wilde and Schwartz (1979), Braverman (1980), and von zur Muehlen (1980).

What we intend to add to this literature is a model of equilibrium price dispersion that is amenable to manipulation. By obtaining an explicit solution, we are able to make comparative-statics predictions and hence to state a number of testable implications. We also derive a rationale for the hypothesis that a firm's demand depends on the difference between its price and the average price in the market. This hypothesis occasionally shows up in an ad hoc fashion in the literature (e.g., Telser 1962; Phelps and Winter 1970; Maccini 1978).

In Section II each firm's demand is expressed as a function of the prices set by all the firms in the market. From this specification, we can see why the lowest-priced firm may not want to raise its price even when total demand is perfectly inelastic and search costs are positive. In Section III, we look more explicitly at each firm's profit-maximizing price decision. We begin with assumptions that there are a finite number of firms, that firms' cost functions may differ, and that there is an array of search costs across consumers. Special cases can be obtained taking limits (e.g., toward an infinity of firms or by collapsing the dispersion of firms' costs).

Section IV goes on to obtain a solution for each firm's equilibrium price as a function of the number of firms, the range and density of consumers' search costs, and the marginal costs of all firms. This, of course, requires some specific assumptions about the distribution of search costs and the firms' cost functions. From those solutions, we can see how changes in key parameters affect the mean and variance of prices. In Section V, we then examine the effects of different kinds of taxes. Section VI contains concluding remarks.

II. Demand Functions

Let there be \( n \) sellers posting prices that are ordered from lowest to highest:

\[
p_1 \leq p_2 \leq \ldots \leq p_n.
\]  

(1)

Suppose consumers enter the market with the following perceived
distribution of prices:
\[
f(p) = 1/n, \quad p = p_1, \ldots, p_n; \\
= 0 \quad \text{otherwise.}
\] (2)

In terms of sampling, the consumer will expect to find any one of the \(n\) prices with equal probability.\(^1\)

Define \(x_k\) as the expected gain from searching for a price lower than \(p_k\). Then
\[
x_k = \sum_{i=1}^{k-1} (p_k - p_i)f(p_i)
\]
\[
= \left[p_k - \sum_{i=1}^{k-1} p_i/(k - 1)\right][(k - 1)/n], \quad k = 2, \ldots, n. \quad (3)
\]

When \(k = 1\), \(x_1 = 0\) because a consumer who finds the lowest price has no expected benefits from additional search. For \(k = 2\), equation (3) becomes \(x_2 = (p_2 - p_1)/n\). In effect, the consumer has one chance in \(n\) of finding \(p_1\) and gaining from search. In general, \(x_k\) is the difference between \(p_k\) and the average of the \(k - 1\) lower prices times the probability of finding one of the lower prices. It can also be verified that if \(p_k = p_{k-1}\), then \(x_k = x_{k-1}\).

In our analysis \(x_k\) is an important variable. It maps the prices posted by firms into a dimension that can be compared directly to the consumer’s cost of search. Given a distribution of search costs, we then have groupings of consumers who will search until they find a price \(p_k\) or better as functions of prices set by all \(n\) firms. This in turn means we can express demand as a function of all prices, thus facilitating an analysis of each firm’s optimal pricing decision.

Consumers may differ in their cost of search \(x\). Assume that the number of buyers with search cost less than or equal to \(x\) can be represented by a continuous function \(G(x)\). Assume \(G(0) = 0\), and let \(G(\infty)\) represent the total number of buyers. Also let \(g(x) = G’(x)\), \(0 < x < \infty\). Reinganum (1979) found that an elastic demand was necessary for equilibrium price dispersion in her model. We shall assume that

---

\(^1\) In Carlson and McAfee (1982), we introduce a perceived distribution for which consumers are not certain about the precise values of the prices that have been set. The continuum of perceived prices in that case helps address a conceptual problem. When the consumer knows the distribution precisely and the number of firms is relatively small, presumably sampling would be without replacement. While the stopping rule can be formulated when sampling is without replacement, we found the subsequent analysis to be analytically intractable. A continuum of perceived prices is more consistent with sampling with replacement, which is implicitly assumed in eq. (3). Because the demand functions are still the same with the more complex perceived distribution, but take considerably more algebra to obtain, we have used the precise perceptions in order to simplify the presentation here.
the quantity demanded by a buyer is completely price inelastic and show that equilibrium price dispersion may still exist when both firms and consumers differ.

A consumer will buy at or below \( p_k \) and not buy at or above \( p_{k+1} \) if and only if \( x_k \leq x < x_{k+1} \). This is depicted in figure 1.\(^2\) Those buyers whose cost of search is below \( x_2 \) will search until they find the lowest price. Those with cost of search below \( x_3 \) but greater than or equal to \( x_2 \) will have an effective reservation price of \( p_2 \). They will terminate search when they find \( p_1 \) or \( p_2 \). At the other extreme, those with a cost of search greater than or equal to \( x_n \) will buy at the first price found.

For reference later, note from (3) that

\[
\frac{\partial x_k}{\partial p_j} = \begin{cases} 
-1/n & j < k \\
(k - 1)/n & j = k \\
0 & j > k. 
\end{cases}
\]  

(4)

An increase in firm \( j \)'s price raises \( x_j \), the critical value of search costs for customers who are marginally willing to accept firm \( j \)'s offer without further search, and may result in some loss of sales to lower-price firms. It also lowers the critical \( x_k \) for higher-price firms so that some customers may then accept a higher price rather than search further.

We now formulate the firm's demand functions. Let \( q_j \) denote expected quantity demanded for the firm that sets price \( p_j \). If each firm is equally likely to be sampled, then \( q_n = (1/n)[G(\infty) - G(x_n)] \). In other words, the highest-priced firm shares equally with all other firms the demand from buyers who do not "shop around" at all.

\(^2\) Figure 1 has been drawn for a uniform distribution in accord with the assumptions used in Sec. IV below, but there is nothing in the definition of \( x_s \) that requires such a shape.
The next-highest-priced firm at \( p_{n-1} \) has the same expected demand as the highest-priced firm plus its share of the demand from customers with \( x_{n-1} \leq x < x_n \): \( q_{n-1} = q_n + \frac{1}{(n-1)}[G(x_n) - G(x_{n-1})] \). Similarly, \( q_j = q_{j+1} + (1/j)[G(x_{j+1}) - G(x_j)] \), where \( j = 1, \ldots, n-1 \).

An alternative form for these demand functions is to write:

\[
q_j = \sum_{k=j}^{n} \frac{1}{k} [G(x_{k+1}) - G(x_k)]
\]

\[
= \frac{G(x_{n+1})}{n} - \frac{G(x_j)}{j} + \sum_{k=j+1}^{n} \left( \frac{1}{k-1} - \frac{1}{k} \right) G(x_k)
\]

or

\[
q_j = \frac{1}{n} G(x_{n+1}) - \frac{1}{j} G(x_j) + \sum_{k=j+1}^{n} \frac{1}{k(k-1)} G(x_k), \tag{5}
\]

where \( G(x_{n+1}) = G(\infty) \).

Equation (5) expresses quantity demanded as a function of the distribution of consumers’ costs of search, and the arguments in those functions depend on the prices set by all firms as shown in (3). To see how a change in price affects a firm’s demand, differentiate (5) with respect to \( p_j \), note that total demand \( G(x_{n+1}) \) is assumed to be unresponsive to price, and use (4) to obtain

\[
\frac{\partial q_j}{\partial p_j} = -\frac{j-1}{jn} g(x_j) - \frac{1}{n} \sum_{k=j+1}^{n} \frac{1}{k(k-1)} g(x_k) \leq 0. \tag{6}
\]

An increase in price will generally result in a loss of business.

This result helps explain why the lowest-price firm may have no incentive to raise its price. Its loss of business comes in the form of customers who have less to gain from seeking out the lowest price. This is a consequence of our assumptions that there are a finite number of firms and that customers’ perceptions of the distribution fully adjust to any changes in price. With an infinity of firms, the probability of sequential searchers finding firm 1 becomes zero, and firm 1 will not lose business when its price is raised.\(^3\)

\(^3\) Burdett and Judd (1979) prove that “a dispersed price equilibrium cannot exist with sequential search if consumers face positive search costs bounded away from zero.” Diamond (1971) is usually credited with being the first to make this point, which is correct within the context of the models postulated. It may not necessarily be true, however, when there are a finite number of firms because of the potential loss of business when the lowest-priced firm considers raising its price.
III. Price-setting Decisions

With customers assumed to be using a sequential, reservation-price, search strategy and to have correct perceptions of the price distribution, firms need to be different in some respects in order to sustain a persistent dispersion of prices. This could take the form of locational advantages or "reputation," which give some firms greater probabilities of being sampled, or of cost differences in supplying the product. We have chosen to develop our model with this latter assumption.

If one thinks about retail goods, such as those studied by PWZ or by Carlson and Pescatrice, differences in costs would have to be associated with labor costs, supplier prices, and costs of space. There may be differences in efficiency in utilizing labor and in deals struck with suppliers, but the major differences within a geographical area are probably associated with costs of using space. In what follows, as in Reinganum's model, it is differences in marginal costs that are critical. This suggests a relatively long-run interpretation of our model if applied to retail sellers. Having chosen a location in the past and finding it more profitable to stay than to move, a firm may have incremental costs of changing the scale of its operation that differ from those situated elsewhere. At the manufacturing level, differences in technology and capital in place can generate differences in short-run marginal costs. Admitting the relevance of many other considerations for actual markets, we shall work out the implications as if the only difference among firms is in their cost functions.\footnote{This in itself is a nontrivial and challenging task. For example, the frequently cited working paper by Arrow and Rothschild (1973) ends with the comment that they had not yet checked whether their analysis goes through when sellers have different costs of production.} Let \( c_j(q_j) \) denote the cost function for firm \( j \).

Suppose the firm's objective is to maximize profits, defined by

\[
\pi_j = p_j q_j - c_j(q_j).
\]  

(7)

The first-order condition for profit maximization is

\[
q_j + [p_j - c'_j(q_j)](\partial q_j / \partial p_j) = 0.
\]  

(8)

The following proposition addresses the question of how the order of prices is related to the order of marginal costs.

PROPOSITION 1: If \( g(x) \) is nondecreasing, then \( c'_1(q_1) \leq c'_2(q_2) \leq \ldots \leq c'_n(q_n) \).

PROOF: Note that

\[
\frac{\partial q_{j+1}}{\partial p_{j+1}} \leq \frac{\partial q_j}{\partial p_j}
\]  

(9)
since, from (6),

\[
\frac{\partial q_j}{\partial p_j} - \frac{\partial q_{j+1}}{\partial p_{j+1}} = - \frac{j - 1}{jn} g(x_j) - \frac{1}{n} \sum_{k=j+1}^{n} \frac{1}{k(k - 1)} g(x_k) \\
- \left[ -\frac{j}{(j + 1)n} g(x_{j+1}) - \frac{1}{n} \sum_{k=j+2}^{n} \frac{1}{k(k - 1)} g(x_k) \right] \\
= \frac{j - 1}{jn} [g(x_{j+1}) - g(x_j)] \geq 0.
\]

Therefore, by (8) and (9),

\[
q_{j+1} + \left[ p_{j+1} - c'_{j+1}(q_{j+1}) \right] \frac{\partial q_j}{\partial p_j} \geq 0. \quad (10)
\]

Subtracting (8) from (10) and rearranging terms, we get

\[
c'_j(q_j) - c'_{j+1}(q_{j+1}) \leq (p_j - p_{j+1}) + (q_j - q_{j+1}) \sqrt{\frac{\partial q_j}{\partial p_j}} \leq 0
\]

as desired.

In the foregoing proof, when \( j = 1 \), we can see that \( \partial q_2/\partial p_2 = \partial q_1/\partial p_1 \). Therefore, for the two lowest-priced firms, \( c'_1(q_1) \leq c'_2(q_2) \) no matter what the shape of the distribution \( G \) of consumers' costs of search. The condition that \( g \) be nondecreasing is sufficient for the firms' marginal costs at their chosen level of output to have the same order as the prices they chose to set. It is not a necessary condition.

Equations (3), (5), and (8) can, in principle, be solved for the \( x_j, q_j, \) and \( p_j \) for \( j = 1, \ldots, n \). In order to obtain an explicit solution, however, we need to make a few simplifying assumptions. This is done in the next section.

IV. An Equilibrium Model

For the cumulative distribution of consumers' search costs, assume

\[
G(x) = x/s, \quad 0 \leq x \leq T; \\
G(x) = T/s, \quad T < x.
\]

Total number of buyers is \( T/s \). The range of search costs is \( T \), and \( 1/s \) is the density for \( 0 \leq x \leq T \).\(^5\) This uniform distribution, of course, has the property of nondecreasing \( g(x) \).

\(^5\) This form facilitates an analysis of the effects of shifting the distribution to the right so that the lowest search costs are strictly positive. That analysis is presented in Carlson and McAfee (1982).
The demand function for firm $j$ now takes the following form:

$$q_j = \frac{1}{sn} \left( T - \frac{n - 1}{n} p_j + \sum_{i \neq j} \frac{p_i}{n} \right) = \frac{1}{sn} [T - (p_j - \bar{p})], \quad (12)$$

where $\bar{p} = \sum_{j=1}^{n} (p_j/n)$. A demand function which depends on the difference between a firm’s price and the average price of all firms is not unusual in the literature. For example, in a Markov model of market shares, Telser (1962) assumes that $p_j - \bar{p}$ influences the transition probabilities from one seller to another. Phelps and Winter (1970) make a similar assumption. Maccini (1978) uses an equation such as (10) in an econometric model. Here this demand function arises as an explicit consequence of sequential search together with a uniform distribution of search costs across buyers.

For later reference, note that

$$\frac{\partial q_j}{\partial p_j} = -\frac{n - 1}{sn^2} < 0. \quad (13)$$

For the firms’ cost functions, we shall use

$$c_j(q_j) = \alpha_j q_j^2 + \beta q_j^3, \quad \alpha_j, \beta > 0; \quad \alpha_j \leq \alpha_{j+1}; \quad j = 1, \ldots, n - 1. \quad (14)$$

The profit function is then

$$\pi_j = p_j q_j - \alpha_j q_j^2 - \beta q_j^3 = (p_j - \alpha_j - \beta q_j)q_j. \quad (15)$$

The first-order condition $\partial \pi_j / \partial p_j = 0$, using (12) and (13), yields

$$(n - 1)(2 + \gamma)p_j = nT(1 + \gamma) + (n - 1)\alpha_j + \sum_{i \neq j} (1 + \gamma)p_i, \quad (16)$$

where

$$\gamma = 2\beta(n - 1)/sn^2. \quad (17)$$

The $n$ equations in (16) may be solved for the $n$ equilibrium prices with the following result:

$$p_j = \alpha_j + \frac{(1 + \gamma)n}{n - 1} \left[ T + \frac{n - 1}{2n - 1 + \gamma n} (\alpha_j - \alpha_j) \right]. \quad (18)$$

---

6 At this point there is a considerable amount of algebra involved in obtaining eq. (12). For those interested, the details are spelled out in Carlson and McAffee (1982). The same can be said for the derivations of eqs. (16) and (18) and the claim that $\pi_j$ is proportional to $q_j^2$.

7 The second-order condition: $\partial^2 \pi_j / \partial p_j^2 = -[(n - 1)(2 + \gamma)/sn^2] < 0$. So the first-order conditions yield a maximum profit for each firm separately given the other firms’ price decisions.
where

$$\bar{\alpha} = \sum_{i=1}^{n} \alpha_i / n. \quad (19)$$

This equation for each firm’s price decision illustrates proposition 1. Since $p_j$ is an increasing function of $\alpha_j$, a firm with a lower marginal cost will set a lower price. We need to check, of course, that every firm earns a positive profit. From (15), we can see that it is necessary that $p_j > \alpha_j$. (It also turns out to be sufficient.) If not, there is no $q_j \geq 0$ that yields positive profits, and $q_i$ is at least zero by the nature of consumer search. For $p_j > \alpha_j$ the term in brackets in (18) must be positive. If that term is positive for the highest-cost firm, then it is positive for all other firms. Consequently, assume

$$\alpha_n - \bar{\alpha} < \frac{2n - 1 + \gamma n}{n - 1} T. \quad (20)$$

If (20) did not hold, then firm $n$ would not stay in the market. Because we are assuming $n$ firms in the market, we may presume that (20) holds. In effect, this determines the number of firms. Given an ordered array of $\alpha_j$, $n$ is the largest $j$ such that (20) holds and $\bar{\alpha}$ is based on all $\alpha_j \leq \alpha_n$.

For any variable $y_j$ define $\text{var } y = \sum_{j=1}^{n} (y_j - \bar{y})^2 / n$. From (18) and (19),

$$\bar{p} = \bar{\alpha} + \frac{(1 + \gamma) n T}{n - 1} \quad (21)$$

and, from (18) again,

$$p_j - \bar{p} = \frac{n - 1}{2n - 1 + \gamma n} (\alpha_j - \bar{\alpha}). \quad (22)$$

Hence

$$\text{var } \bar{p} = \left( \frac{n - 1}{2n - 1 + \gamma n} \right)^2 \text{var } \alpha. \quad (23)$$

We can now readily show how changes in $s$, $\beta$, and $n$ will affect $p_j$ and $\text{var } \bar{p}$. From (18),

$$\frac{\partial p_j}{\partial \gamma} = \frac{n}{n - 1} \left[ T + \frac{(\bar{\alpha} - \alpha_j)(n - 1)^2}{(2n - 1 + \gamma n)^2} \right] > \frac{n^2 T(1 + \gamma)}{(n - 1)(2n - 1 + \gamma n)}$$

by (20). From (23),

$$\frac{\partial \text{var } \bar{p}}{\partial \gamma} = - \frac{2n(n - 1)^2}{(2n - 1 + \gamma n)^3} \text{var } \alpha < 0.$$
The parameter $\gamma$, defined in (17), is increased by an increase in $\beta$ (steeper marginal cost curves) and by a decrease in $s$ (increase in demand density). Thus, higher $\beta$ or lower $s$ will raise the price of every firm in the market but will have relatively more effect on the lower-cost firms, and hence the variance of prices will fall.

The effect of a change in the number of firms on the variance of prices can be seen by substituting $\gamma = 2\beta(n - 1)/sn^2$ into (23):

$$\text{var } p = \left(\frac{1}{2 + [1/(n - 1)] + (2\beta/sn)}\right)^2 \text{var } \alpha. \quad (24)$$

What happens to $\text{var } p$ if there were more firms but with the same $\text{var } \alpha$? Somewhat surprisingly, the answer is $\text{var } p$ increases. This can be seen by noting that, if $\beta \geq 0$, the term in brackets increases as $n$ increases. This increase, however, is bounded from above. Specifically, $\text{var } p < \frac{1}{4} \text{var } \alpha$ and $\lim_{n \to \infty} \text{var } p = \frac{1}{4} \text{var } \alpha$. In this model, the variance of marginal costs limits the variance of prices. The density, but not the range of search costs, plays a minor role, and even that effect disappears if $\beta = 0$. MacMinn (1980) has a coefficient of $\frac{1}{4}$ in a model with a continuum of firms, which is what this model predicts as $n \to \infty$. In our model, however, that limit rises when we introduce a proportional tax in Section V.\(^8\)

Another feature of this model is that lower-cost firms are more profitable. More precisely, $\pi_j$ is proportional to $q_j^2$.

V. Cost Increases and Taxes

If all $\alpha_j$ increase by $\Delta \alpha$, the effect as seen in (18) is that $p_j$ rises by $\Delta \alpha$. All such cost increases get completely passed on.

If firm $j$'s cost increase is greater than the average cost increase, then firm $j$'s price will rise by more than the average price but not by as much as its cost increase. This can be seen directly from (22), since $0 < \left(\frac{(n - 1)(2n - 1 + \gamma n)}{(n - 1)}\right) < \frac{1}{2}$ when $\gamma \geq 0$ and $n \geq 2$.

In terms of tax levies, it makes a difference whether the tax is per unit or proportional. A per unit tax is like a cost increase for all firms. It is completely passed on in higher prices and the variance of prices is unchanged.

Suppose, however, a proportional tax ($0 < \tau < 1$) is put on the selling price. Firm $j$ maximizes $\pi_j = [p_j(1 - \tau) - \alpha_j - \beta q_j]q_j$ or, equivalently for a fixed $\tau$, the firm maximizes $\pi_j^* = (p_j^* - \alpha_j^* - \beta^* q_j)q_j$, where $p_j^*$ is the new price, $\alpha_j^* = \alpha_j/(1 - \tau)$, and $\beta^* = \beta/(1 - \tau)$.

\(^8\) The variance of prices is also increased when search costs all rise, as shown in Carlson and McAfee (1982), although if the increase in search costs is large enough, equilibrium price dispersion breaks down.
This has the same form as (15) but with the cost parameters adjusted for the tax effect. Let $\gamma^* = \gamma/(1 - \tau)$. If $\bar{p}^*$ denotes the new average price, then by (21),

$$\bar{p}^* = \frac{\bar{\alpha}}{1 - \tau} + \frac{(1 + \gamma^*)nT}{n - 1}.$$ 

The average price received by firms after taxes is

$$(1 - \tau)\bar{p}^* = \bar{\alpha} + \frac{(1 - \tau + \gamma)nT}{n - 1} = \bar{p} - \frac{\tau nT}{n - 1} < \bar{p}.$$ 

Therefore, the firms do not succeed in passing on all of a proportional tax, even though demand has been assumed to be perfectly price inelastic. The implication of this result is that, if the tax revenues were the same, the consumer would prefer a proportional tax to a per unit tax.

In terms of equation (23) for $\var{p}$, the introduction of a proportional tax has the effect of raising $\var{\alpha}$ and lowering slightly its coefficient. The net effect is that the variance of prices does increase. Note first that

$$\var{p^*} = \frac{(n - 1)}{2n - 1 + [n\gamma/(1 - \tau)]^2} \cdot \frac{\var{\alpha}}{(1 - \tau)^2},$$

$$= \frac{(n - 1)^2 \var{\alpha}}{[(2n - 1)(1 - \tau) + n\gamma]^2}.$$ 

From this it is evident that, as $\tau$ increases, so does $\var{p^*}$.

A few years ago the state of Indiana changed its gasoline tax from per gallon to a percentage of the price. This model suggests that the incidence of the tax could have been affected by that change. Another prediction is that the variance of gasoline prices would be increased by this change.

VI. Concluding Remarks

Our primary intent has been to develop a rigorous equilibrium price dispersion model which produces testable predictions. We started with the presumption that in many markets there are a finite number of firms, firms differ in their cost functions, and consumers differ in their search costs. Results when there is a continuum of firms can be found as special cases by taking limits. A number of specific assumptions about the distribution of search costs and about the nature of firms' cost functions enable us to find explicit solutions within a general equilibrium framework. This helps us see more clearly the nature
of the interrelationships and to obtain clear-cut predictions. A number of these predictions, or potential empirical tests, are

1. Lower-cost firms tend to set lower prices and have greater quantity demanded.

2. As a result of the search by consumers, demand may be a linear function of the difference between a firm's price and the average price set by all firms.

3. Profits are proportional to the square of quantity demanded (when search costs are distributed uniformly from zero to some upper bound).

4. The number of firms is influenced by the distribution of cost functions of potential firms, and, ceteris paribus, there will be more firms the larger the range of consumer search costs.

5. The variance of prices varies directly with the variance of costs (as measured by the intercept terms in the firms' marginal cost functions).

6. Ceteris paribus, the variance of prices is increased by an increase in the number of firms, by a decrease in the slope of the marginal cost functions, and by a decrease in the density of the distribution of consumer search costs.

7. A per unit tax will be completely passed on with no change in the variance of prices, when demand is perfectly inelastic. A proportional tax will not be fully passed on and will increase the variance of prices.

Considerations other than those introduced in this model undoubtedly contribute to observed price dispersion. An elastic demand reinforces incentives for the lowest-cost firm to price below competitors. Different probabilities of being sampled and perceived differences in quality of products, whether real or imagined, influence pricing decisions. Differential abilities to alter the sampling probabilities and perceptions of quality through advertising or other means could be worth analyzing. And the use of periodic sales, as shown by Varian (1980), will add to observed price dispersion. As these dimensions are integrated and developed more rigorously and the resulting predictions subjected to empirical testing, our understanding of the role of imperfect information and competitive advantages in observed pricing structures will be enhanced.

References


