Review of Economic Studies (2008) **75**, 317–332 0034-6527/08/00130317$02.00
© 2008 The Review of Economic Studies Limited

Capacity Choice Counters the Coase Conjecture

R. PRESTON MCAFEE
California Institute of Technology
and
THOMAS WISEMAN
University of Texas at Austin

First version received August 2005; *final version accepted March* 2007 (*Eds.*)

The Coase conjecture (1972) is the proposition that a durable-goods monopolist, who sells over time and can quickly reduce prices as sales are made, will price at marginal cost. We show that an arbitrarily small deviation from Coase's assumptions—a deviation that applies in almost any practical application—results in the failure of that conjecture. In particular, we examine that conjecture in a model where there is a vanishingly small cost for production (or sales) capacity, and the seller may augment capacity in every period. In the "gap case", any positive capacity cost ensures that in the limit, as the size of the gap and the time between sales periods shrink, the monopolist obtains profits identical to those that would prevail when she could commit *ex ante* to a fixed capacity. Those profits are at least 29·8% of the full static monopoly optimum.

1. INTRODUCTION

Nobel laureate Ronald Coase (1972) startled the economics profession with a counterintuitive proposition, which came to be known as the Coase conjecture, concerning the monopoly seller of a durable good. Coase's original example was the hypothetical owner of all land in the U.S. The monopolist maximizes profits by identifying the monopoly price and selling the quantity associated with that price. Having sold that quantity, however, the monopolist now faces a residual demand, and she is induced to try to sell some additional units to the remaining buyers at a price that is lower than the initial price. Such logic entails a sequence of sales at prices falling towards marginal cost. Rationally anticipating falling prices causes most potential buyers to wait for future lower prices. Provided that the monopolist can make sales and cut price sufficiently rapidly, Coase conjectured that the monopolist's initial offer would be approximately marginal cost and that the monopoly would replicate the competitive outcome.

Intuitively, the monopolist competes with future incarnations of herself. Even when facing a monopolist, buyers have an alternative supplier: the monopolist in the future. That the power of such a substitution possibility might render the monopoly perfectly competitive remains a captivating idea even for an audience accustomed to the fact that subgame perfection (or time consistency) restricts equilibria in dramatic ways. Arguably, Coase's conjecture remains the most extreme example of the power of subgame perfection.

Coase's intuition is compelling, but its consequence—that a monopolist makes no profit—is difficult to accept. We offer a way to reconcile this paradox in the form of a countervailing intuition that Coase did not consider. In almost any real-world application, there is a small cost of capacity, so that selling a given amount over a shorter span of time costs more. Even Coase's hypothetical seller of all U.S. land, who faces no production cost, still bears a capacity cost: to sell the land rapidly requires a large number of sales agents, so the seller must incur hiring and training costs. Whenever there is an increased cost of increased speed, there is an effective capacity cost.[1] In that case, we show that even if the seller can augment capacity at any time, she can credibly commit to the same level that she would pick if the choice were once and for all. By doing so, she earns at least 29·8% of the static monopoly profit, instead of the zero profit predicted by Coase's argument.

The first formal proofs of the Coase conjecture are given by Bulow (1982) and Stokey (1982). The deepest analysis is the challenging paper by Gul, Sonnenschein and Wilson (1986). This paper distinguishes two cases, the so-called "gap" case, where the lowest value buyer has a value strictly exceeding marginal cost, and the "no-gap" case, in which demand and marginal cost intersect. The gap case, which is also studied by Fudenberg, Levine, and Tirole (1985), is more readily analysed because there is generically a unique subgame perfect equilibrium. Uniqueness arises because if there are few potential buyers left, it pays to sell to all of them at the lowest willingness to pay (which strictly exceeds marginal cost by hypothesis). This conclusion ties down the price in the last stage and ensures that the game is of finite length; backwards induction then gives the unique equilibrium. When the stages occur rapidly, prices converge to the final price rapidly, so buyers are unwilling to pay much more than the lowest valuation, which implies that the opening price is the lowest valuation. That result is not quite the same as Coase's conjecture because prices converge on the lowest willingness to pay rather than to marginal cost, but it is similar in spirit.

In contrast, in the "no-gap" case, the Coase conjecture holds in some equilibria, but not in others. There is a "Coasian" equilibrium that is stationary (at any time, buyers' strategies depend only on the current price and not upon the prior history of the game) and entails an initial price close to marginal cost. Moreover, this opening price converges to marginal cost as the time between sales periods approaches zero. As Ausubel and Deneckere (1989) demonstrate, this Coasian equilibrium can be used to ensure the existence of other, non-stationary equilibria by threatening the seller that, should she deviate from the hypothesized equilibrium, buyers' beliefs will revert to the Coasian equilibrium, which involves low profits for the seller. Such a threat guarantees that the seller would not deviate from anything at least as profitable as the low-profit Coasian equilibrium, and seller profits anywhere up to the full static monopoly profit can be obtained in equilibrium.

We focus on the gap case, and show that the Coase conjecture is not robust to the introduction of endogenous, costly capacity, a modification that would be relevant in nearly all practical settings. We envision a perfectly durable capacity, so that, once bought, the capacity never needs to be repurchased. If a monopolist chose production capacity at the beginning of time and could not augment it later, she would use capacity as a commitment device, setting a low capacity and dribbling output into the market. That approach has the advantage of ensuring that prices are high early and fall slowly, as high-value buyers pay more for early acquisition of the good. Indeed, we will demonstrate that in such a "commitment game", the monopolist obtains at least 29·8% of the static monopoly profits, no matter how fast the stages of the game occur; increasing the speed of the game induces the monopolist to cut capacity in a way that keeps the flow of goods to buyers constant. Note that the monopolist could not achieve anything higher than static monopoly profits even if she could commit *ex ante* to a sequence of prices. Stokey (1979) shows that she would optimally set the static monopoly price in each period and thus earn static monopoly profits. That is, the ability to discriminate dynamically does not help the monopolist.

1. A constant marginal cost of selling does not create a capacity cost, but just an ordinary marginal cost. Potential congestion in sales, so that selling twice as fast incurs more than twice the costs, creates an effective capacity cost.

© 2008 The Review of Economic Studies Limited

While the specificity of the lower bound of profits may be remarkable, the fact that profits fail to converge to zero is not; such a model endows the seller with an extraordinary commitment ability—the ability to commit at the beginning of the game to restrict future sales. This seller does not reach the full static profits because she cannot stop herself from selling to all customers eventually; when she reaches the monopoly quantity, Coasian logic dictates that she continue to sell. However, she can slow herself down, selling slowly enough to ensure that she acquires a significant fraction of the monopoly profits. She loses two ways relative to the static monopoly—she eventually sells too much and profits are earned slowly and hence discounted, but nevertheless capacity commitment leads to positive profits.

To add capacity in a more realistic fashion, we consider a monopolist who chooses whether, and by how much, to augment production capacity in each period of the game. The monopolist then sets a price and sells the lesser of the demand by buyers and the production capacity. Suppose that the monopolist faces a small cost of capacity. By Coase's logic, she is tempted to cut prices as quickly as possible by shrinking the time between sales periods. Holding the cost fixed, the surprising fact is that when the gap is small, then in the equilibrium of the game with augmentable capacity, the seller can choose the same capacity and earn the same profits as in the game in which capacity cannot be increased. That is, the ability to increase capacity later does not harm the seller even in a Coasian environment, for equilibrium profit levels are the same as those that arise when capacity is chosen once and for all. Consequently, *any* positive cost of capacity prevents the opening price from approaching marginal cost. The presence of a capacity cost permits the seller to behave as if she could commit to capacity initially. In contrast, with a zero cost of capacity, the game is strategically equivalent to the one that Gul *et al.* (1986) study: only the Coase equilibrium, with zero profits in the limit, occurs. Thus, there is a discontinuity in seller profits as the cost of capacity goes to zero.

The intuition behind the theorem suggests an effect that Coase's reasoning neglects. The Coasian price path requires a seller to sell to the whole market very rapidly. Because buyers will wait for prices close to marginal cost (since these are coming rapidly), the opening price is close to marginal cost, and most sales take place in the first few minutes. In the limit as the sales periods get arbitrarily close, all sales take place immediately. In environments requiring production or some transaction medium, that outcome requires the seller to produce a very large production facility or high-bandwidth transaction facility, so that the flow of sales can be extremely large for a very short period of time. If the cost of capacity is high relative to the size of the market, then the seller will not purchase so much capacity. That observation means that for any positive cost, no matter how small, the seller will not increase capacity near the end of the game, when few buyers remain, which in turn allows her to credibly commit to a low level of capacity at the beginning. The logic of backwards induction compels buyers to believe that she will not increase capacity in the future, and thus that prices will fall slowly. Subgame perfection, which leads to the Coase conjecture by forcing the monopolist to lower prices as quickly as possible, now offers her a credible way to delay sales.

Another way to see that intuition is as follows. We will show that in the "commitment game", where the seller chooses capacity once and for all at the beginning, the optimal capacity increases with the size of the market. As sales are made, the desired commitment capacity falls. Thus, a seller who chooses a starting capacity in a neighbourhood of the initial optimal level of capacity will not be later tempted to increase it because the starting capacity will still exceed the subsequent lower desired level. That result means that the seller has local commitment ability—she can effectively commit to a slight reduction or increase in capacity around the optimal opening level. But the ability to vary capacity locally around the global optimum is sufficient to ensure that profits are maximized as a function of capacity because the first-order conditions are satisfied.

© 2008 The Review of Economic Studies Limited

That is, the level of profits when capacity is augmentable is identical to that when capacity is chosen once and for all; the local maximum is the same as the global.

Our result that capacity choice in each period delivers the same profits as the commitment version holds in the limit of the gap case, as the gap shrinks to zero. That case, where the gap is positive but small, had been the only setting left where the Coase conjecture had bite and the monopolist made no profit. In the no-gap case, Ausubel and Deneckere (1989) show the existence of equilibria where the seller makes high profits. In the gap case, the monopolist sells at a price near the lowest consumer's valuation, but if the gap between marginal cost and the lowest valuation is large, then that price entails high profits. In this paper, we show that the seller can earn substantial profits even when the gap is vanishingly small. Thus, in any reasonable economic situation Coase's conclusion does not hold: a durable-goods monopolist *can* make profits.

Alternatively, our result can be interpreted as a way to select from the continuum of equilibria of Ausubel and Deneckere (1989) in the no-gap case: when the monopolist chooses costly capacity, equilibria where the monopolist makes very low profits are not robust to the introduction of a small discontinuity of buyers' valuations just above marginal cost. Note that without capacity choice, the selected equilibrium is very different. With a very small gap, the unique equilibrium entails very low profits.

There has been some earlier work on the effect of capacity in the Coase setting. Kahn (1986) offers a model in which more rapid sales cost more, recognizing the restrictions on the seller emphasized by this paper. Kahn's model features a quadratic cost of the rate of sales. The increasing cost of faster selling ensures that the equilibrium involves positive profits. Our result makes two contributions relative to Kahn's work. First, we endogenize the seller's capacity; Kahn treats it as an exogenous parameter. Second, in the limit as the cost shrinks, Kahn's seller is again making zero profits (the Coase outcome), while in our model the seller continues to make a substantial fraction of static monopoly revenue, a fraction which remains positive (and greater than 0·298) even in the limit. We conjecture, however, that a version of our results (in particular, that *ex ante* commitment to selling capacity produces the same profits as augmentable selling capacity) holds in a generalization of Kahn's model with endogenized capacity. Moreover, there is no obvious impediment to employing an analogous proof, where we interpret capacity as an input that reduces the slope of the quadratic cost, but further investigation is needed. The thought experiment of the present paper is very natural in Kahn's elegant framework. Intuitively, Kahn's cost function is a smoothed version of ours—in our model, the production cost in a sales period is zero for quantities below capacity and infinite beyond it.

Closer to our result, Bulow (1982) presents a very clean, parameterized example of the no-gap case with the property that the optimal commitment capacity is invariant to the size of the market, and constructs an equilibrium with costless capacity in which the seller initially chooses that optimal level and is never tempted to increase it. In light of subsequent work of Ausubel and Deneckere (1989) showing the multiplicity of equilibria in the no-gap case, the existence of an equilibrium in which the seller earns positive profits is less surprising, and Bulow does not show that the constructed equilibrium is unique or that there is a lower bound on the seller's equilibrium profits. Further, he notes that in the gap case his equilibrium would unravel. Nevertheless, the original insight that capacity choice might limit the Coase conjecture is due to Bulow. In fact, he speculates that introducing a cost of capacity might restore his equilibrium in the gap case.

The rest of the paper proceeds as follows. In Section 2, we set up the model. In Section 3, we present the first main result: in the version of the game with once-and-for-all capacity choice, the seller can earn at least 29·8% of static monopoly profits. Section 4 contains the other main theorem, showing that the outcome with capacity choice in each period mirrors the outcome with initial capacity commitment. In Section 5, we conclude.

© 2008 The Review of Economic Studies Limited

2. MODEL

A durable-good monopolist faces a market in which a continuum of consumers, indexed by $q \in [0, q_0]$, each demand a single unit. Both consumers and the monopolist live forever and discount the future at the rate r. Sales can occur at discrete, equally spaced intervals. The time between such sales periods is Δ. Thus, period z occurs at time $z\Delta$, and the discount rate per period is $e^{-r\Delta}$. In order to produce the good, the monopolist must invest in capacity. The cost of buying the capacity to produce (or to sell) at a constant flow rate of one unit of the good per unit of time is c, which implies that the capacity to produce one unit per *sales period* costs c/Δ. Capacity can be purchased in any amount—that is, it is a continuous decision variable for the seller. There is no depreciation: once purchased, capacity is good forever. Furthermore, there are no other production costs. If the monopolist has capacity K, she can sell at a rate of K units per time period at zero marginal cost forever. Consumers' valuations are determined as follows.

The value of a unit of the good to consumer q is given by $v(q) \equiv p(q) + g$, where the constant g is strictly greater than zero and p is a decreasing, twice-differentiable function from $[0, q_0]$ to $\boldsymbol{R}_+$ such that $p(q_0) = 0, p(q) > 0$ for $q < q_0$, and $p'(q) + qp''(q) < 0$ for all q. (The property that $p'(q) + qp''(q) < 0$ is a standard regularity condition. It is equivalent to log concavity of demand, which ensures, for example, that the best response functions of Cournot duopolists are downwards sloping and that a monopolist facing a per-unit tax increases his price by less than the amount of the tax.)[2] Consumers' valuations are bounded above by $v(0)$ and below by $g(= p(q_0) + g)$. This is the "gap case", where the lowest valuation among the buyers is strictly greater than the monopolist's marginal cost. The revenue function $R(q)$ is given by $R(q) = qv(q)$, and marginal revenue $\mathrm{MR}(q)$ is $qv'(q) + v(q)(= qp'(q) + v(q))$. Setting $\mathrm{MR}(q_m) = 0$ defines the static monopoly quantity q_m.

At the beginning of each sales period, the monopolist publicly chooses how much additional capacity to purchase. She then announces a price P for that period, and must sell to any buyer who wants to buy at that price, up to a maximum of $K\Delta$ units. (The order of the monopolist's choices of price and capacity does not matter, as long as both are chosen before buyers move.) We will assume that the rationing rule is to serve higher valuation buyers first.[3]

Assumption 1. *If the quantity of consumers who wish to buy in any sales period z is greater than $K_z\Delta$, then sales will be made to the subset of size $K_z\Delta$ of potential buyers with the highest valuations.*

The goal of the monopolist is to maximize the discounted value of revenue minus the discounted value of expenditures on capacity. (Typically, we will be considering vanishingly small costs of capacity, so that revenue and profits are almost interchangeable.) The consumers seek to maximize their discounted surplus. The surplus to consumer q who buys in sales period z at price P_z is $e^{-rz\Delta}[v(q) - P_z]$. As is standard in the literature on the Coase conjecture, we will consider only equilibria where deviations by a zero mass set of consumers have no effect on continuation play.

In the absence of capacity constraints, Fudenberg *et al.* (1985) and Gul *et al.* (1986) show that there is generically a unique subgame perfect equilibrium in the gap case. That equilibrium satisfies the Coase conjecture, in the sense that as the period length Δ goes to zero, the

2. We note that the set of functions satisfying that condition is closed under truncation: since $p'(q) < 0, p'(q) + (q - -x)p''(q) < 0$ for all $x \in [0, q_0)$ and all $q \in [x, q_0]$. We note also that the regularity condition is not required for the results of Fudenberg *et al.* (1985) and Gul *et al.* (1986).

3. Assumption 1 is made for the sake of tractability. We note, though, that along the equilibrium path no rationing will occur. Denicolo and Garella (1999) study the effect of assuming random rationing rather than efficient rationing in a two-period Coase model.

© 2008 The Review of Economic Studies Limited

monopolist earns profits close to gq_0 by setting an initial price close to g and selling to the entire market nearly instantaneously. In that equilibrium, which we will call the Coase equilibrium, all consumers are served in a finite number of sales periods, which implies that prices in each period can be determined by backwards induction.

When there are no capacity constraints, the equilibrium path has the "skimming" property. That is, in any period there is a cut-off valuation $\bar{\upsilon}$ such that all consumers with valuations greater than $\bar{\upsilon}$ have already bought, and all consumers with valuations less than $\bar{\upsilon}$ have yet to buy (see Fudenberg *et al.*, 1985, Lemma 1; Ausubel and Deneckere, 1989, Lemma 2.1). The intuition is that if a consumer with valuation υ is willing to buy at price P, then so is any consumer with valuation $\upsilon' > \upsilon$. Both buyers get the same benefit (in the form of lower future prices) from waiting, but the cost of delaying consumption is greater for the high-valuation consumer. Thus, in any period the remaining market can be characterized completely by x, the volume of consumers who have been served so far.

In principle, the skimming property may fail when we introduce capacity constraints because of the rationing rule that favours high-valuation customers. Even if both types of consumers would prefer to wait and buy at tomorrow's prices, the low-valuation type might still purchase today if he knows that he would be rationed out of the market tomorrow. However, that situation does not arise in our equilibrium analysis—it requires consumers to believe that rationing will occur, but in any period where buyers are rationed the seller could increase revenue by raising the price without affecting the quantity sold. For ease of exposition, therefore, we will continue to let x_z denote the volume of consumers who have already been served at the beginning of period z. Note that if the size of the remaining market is such that sales in each period of the corresponding Coase equilibrium are no greater than $K\Delta$, then the capacity constraint will never bind. In that case, capacity is no longer relevant, and the only subgame perfect continuation is the Coase equilibrium.

We obtain two main results. First, the optimal once-and-for-all capacity choice gives the seller at least 29·8% of static monopoly revenue. Second, even if the seller can augment capacity at any time, she can still credibly commit to the same, optimal level, and thus must earn at least the same profits in any subgame perfect equilibrium. Our strategy of proof for the second result is the following. To begin, instead of directly examining the situation that we are interested in, we look at the continuous time case with no gap, *assuming* that the monopolist sells up to capacity at every instant and that she can costlessly choose capacity once and for all at the start of the game. In that case, we show that the optimal commitment capacity is decreasing in the quantity of buyers already served and that commitment profits are concave in capacity. Those properties also hold (maintaining the two assumptions) in the limit of the discrete time case with a small gap, as the length of the periods shrinks. Next, we demonstrate that the two assumptions are results when we consider the subgame perfect outcomes of the discrete time game in which the seller can augment capacity in any period. In particular, if the seller chooses the optimal once-and-for-all level of capacity in the first period, then in the subgame perfect continuation she will never increase it, and sales will equal capacity in every period (except possibly for a few periods at the end of the game). As an immediate consequence, the seller must make at least 29·8% of static monopoly revenue in any subgame perfect equilibrium, no matter how small is the (strictly positive) cost of capacity.

3. ONE-TIME CAPACITY CHOICE

In this section, we consider the case of one-time capacity choice. Suppose that the monopolist can only purchase capacity at the beginning of the game. If she purchases none, the game is over. For non-zero K, let $\rho_{\Delta}^{\text{com}}(K, g)$ be the seller's revenue in subgame perfect equilibrium from

© 2008 The Review of Economic Studies Limited

committing to capacity K when the gap is g. (An argument similar to Lemma 3 of Fudenberg *et al.*, 1985, shows that for any positive gap g, the monopolist will sell to all consumers in finite time, and so generically there is a unique subgame perfect equilibrium solvable through backwards induction.) Let $K_{\Delta}^{\text{com}}(g)$ be the capacity that maximizes commitment revenue $\rho_{\Delta}^{\text{com}}(K, g)$, and let $\mathrm{P}_{\Delta}^{\text{com}}(g)$ be the value of that maximized revenue.

As the time between offers Δ shrinks to zero, how large is the maximized commitment revenue $\mathrm{P}_{\Delta}^{\text{com}}(g)$ relative to the static monopoly profits $R(q_m)$? (Remember that the static monopoly profit is the highest that the monopolist could attain even with the ability to commit to future prices.) Theorem 1 provides a global minimum for revenue equal to a fraction γ of the static monopoly profits. The exact value of γ, which approximately equals 0·298425, is defined as follows.

Definition 1. Define the constant γ as the value $\max_{x>0} \frac{1-e^{-x}(1+x)}{x}$.

Theorem 1. *As Δ shrinks to zero, the limit maximized commitment revenue* $\lim_{\Delta \to 0} \mathrm{P}_{\Delta}^{\text{com}}(g) \geq \gamma R(q_m)$.

Proof.[4] We manipulate the revenue expression to derive the lower bound on revenue. The seller's equilibrium revenue $\mathrm{P}_{\Delta}^{\text{com}}(g)$ is no less than the revenue $\bar{\mathrm{P}}_{\Delta}^{\text{com}}(g)$ that she would obtain if she were restricted to offer prices so as to sell quantity equal to capacity $K\Delta$ in each period (except possibly in the last sales period, if the remaining size of the market is less than $K\Delta$). The market is saturated at $T = \lceil q_0/K\Delta \rceil$, the smallest integer at least as great as $q_0/K\Delta$; at this point, the price is g. Let υ_t be the value of the marginal buyer (*i.e.* the one with the lowest valuation) in period t (so that $\upsilon_t = \upsilon(K\Delta t)$ for $t < T$). Let P_t be the price charged by the seller. Since $P_T = g$, and the marginal buyer in any earlier period t is indifferent between buying in period t and in period $t+1$, we have

$$
\begin{aligned}
&P_T = \upsilon_T = g, \text{ and} \\
&P_t = (1 - e^{-r\Delta})\upsilon_t + e^{-r\Delta} P_{t+1}, \text{ so} \\
&P_t = e^{-r\Delta(T-t+1)} g + (1 - e^{-r\Delta}) \sum_{j=0}^{T-t} e^{-r\Delta j} \upsilon_{t+j}. \qquad (1)
\end{aligned}
$$

The quantity sold q in each period before T is $K\Delta$, so substituting expression (1) into the formula for revenue yields

$$
\begin{aligned}
\bar{\mathrm{P}}_{\Delta}^{\text{com}}(g) &= \sum_{t=1}^{T} e^{-r\Delta(t-1)} P_t q_t \\
&\geq -e^{-r\Delta(T-1)} gK\Delta + \sum_{t=1}^{T} e^{-r\Delta(t-1)} (1 - e^{-r\Delta}) \left(\frac{e^{-r\Delta(T-t+1)}}{(1 - e^{-r\Delta})} g + \sum_{j=0}^{T-t} e^{-r\Delta j} \upsilon_{t+j} \right) K\Delta \\
&\geq -e^{-r\Delta(T-1)} gK\Delta + (1 - e^{-r\Delta}) K\Delta \sum_{t=1}^{T} \left(e^{-r\Delta(t-1)} \upsilon_t t + \frac{e^{-r\Delta T}}{(1 - e^{-r\Delta})} g \right) \\
&\geq -e^{-r\Delta(T-1)} gK\Delta + (1 - e^{-r\Delta}) K\Delta \sum_{t=1}^{T} e^{-r\Delta(t-1)} \upsilon_t t, \qquad (2)
\end{aligned}
$$

4. We thank a referee for suggesting this proof, which is much simpler than our original version.

© 2008 The Review of Economic Studies Limited

where the term $-e^{-r\Delta(T-1)}gK\Delta$ reflects the possibility that sales in the last period may be less than $K\Delta$. Now, let $m = \lceil q_m/K\Delta \rceil$, the number of periods it takes to sell the monopoly quantity q_m. Since $m \leq T$ and υ_t decreases with t,

$$\begin{aligned}\bar{\mathrm{P}}_{\Delta}^{\mathrm{com}}(g) &\geq -e^{-r\Delta(T-1)}gK\Delta + (1-e^{-r\Delta})K\Delta\sum_{t=1}^{m} e^{-r\Delta(t-1)}\upsilon_m t \\ &= -e^{-r\Delta(T-1)}gK\Delta + (1-e^{-r\Delta})K\Delta\upsilon_m \frac{1-e^{-r\Delta m}(1+(1-e^{-r\Delta})m)}{(1-e^{-r\Delta})^2} \\ &= -e^{-r\Delta(T-1)}gK\Delta + K\Delta m\upsilon_m \frac{1-e^{-r\Delta m}(1+(1-e^{-r\Delta})m)}{(1-e^{-r\Delta})m}.\end{aligned}$$

As Δ shrinks, $K\Delta m$ approaches q_m, and thus υ_m approaches the monopoly price $p(q_m)$, so $K\Delta m\upsilon_m$ converges to monopoly profits $R(q_m)$. In addition, the term $-e^{-r\Delta(T-1)}gK\Delta$ shrinks to zero with Δ. Thus, defining z as $(1-e^{-r\Delta})m$, we can write

$$\begin{aligned}\lim_{\Delta\to 0}\frac{\bar{\mathrm{P}}_{\Delta}^{\mathrm{com}}(g)}{R(q_m)} &\geq \max_{m\geq 1}\left(\frac{1-e^{-r\Delta m}(1+(1-e^{-r\Delta})m)}{(1-e^{-r\Delta})m}\right) \\ &= \max_{z\geq 1-e^{-r\Delta}}\left(\frac{1-e^{z\left(\frac{-r\Delta}{1-e^{-r\Delta}}\right)}(1+z)}{z}\right).\end{aligned}$$

Since $-r\Delta/1-e^{-r\Delta}$ is decreasing in Δ and converges to -1 as Δ shrinks to zero (the proof is in the Appendix), we get

$$\lim_{\Delta\to 0}\frac{\bar{\mathrm{P}}_{\Delta}^{\mathrm{com}}(g)}{R(q_m)} \geq \max_{z\geq 1-e^{-r\Delta}}\frac{1-e^{-z}(1+z)}{z} = \gamma.$$

(The constraint does not bind because the maximizer is roughly 1·79.) ‖

That bound, which is independent of the interest rate r, is tight. Suppose that $p(q) = (1-q)^{\alpha}$, for α near zero. This functional form satisfies the demand conditions. As α goes to zero, demand converges to "unit" demand, so m approaches T and υ_t approaches υ_m for all t. At the other extreme, the profits with a capacity constraint may be arbitrarily close to the static monopoly profits. In the constant elasticity of demand case, in the limit as the elasticity converges to 1, the ratio of the capacity-constrained revenue to the static monopoly profits converges to 1 (when time is continuous).[5] We note that expression (2) can be rewritten as the sum of a few terms that shrink to zero with Δ plus

$$\sum_{t=1}^{T-1} e^{-r\Delta t}[\upsilon_{t+1}K\Delta(t+1) - \upsilon_t K\Delta t].$$

Thus, commitment revenue is roughly equal to the discounted sum of buyers' static marginal revenues, which illustrates the seller's twofold loss relative to static monopoly. First, she eventually sells beyond the monopoly quantity q_m to buyers with negative marginal revenues. Second, she sells to the high-value consumers (those who have positive marginal revenues) only gradually, leading to delay costs. Those losses are proportionally greatest when the monopoly quantity q_m is large (*i.e.* close to q_0), as it is when $p(q) = (1-q)^{\alpha}$ for low α. It takes time to sell to all the high-value consumers, and the wait between selling to the last high-value consumer and reaching price zero at q_0 is very short. In the case of constant elasticity close to 1, conversely, q_m is close

5. The derivation is available from the authors upon request.

© 2008 The Review of Economic Studies Limited

to zero, which minimizes the loss. Linear demand $p(q) = 1 - q$ lies in between. Numerical computation shows that for linear demand, profits are roughly 55·74% of the static monopoly profits of 0·25 (again with continuous time). At an annual interest rate of 5%, the monopolist optimally sells to approximately 1·86% of the market per year.

Next, as a preliminary to Theorem 2, we examine how the monopolist's optimal level of commitment capacity changes as the size of the remaining market shrinks. Let $\rho_{\Delta}^{\text{com}}(K, x, g)$ be the seller's revenue in subgame perfect equilibrium from committing to capacity K when quantity x has already been sold and the gap is g. We study $\rho_{\Delta}^{\text{com}}(K, x, g)$ indirectly by considering the limiting, continuous time case with no gap, *assuming* that the monopolist sells at a rate equal to capacity until demand is satisfied. In that case, what if, after having sold quantity x, the seller were given a one-time chance to adjust her capacity? Define $\rho^{\text{com}}(K, x)$ as the revenue remaining when quantity x has been sold and K is the capacity, and let the optimal capacity be $K^{\text{com}}(x)$. Lemma 1 shows that the optimal capacity decreases with x, and that commitment profit is strictly concave in capacity. That is, as the size of the market shrinks, the monopolist prefers to decrease capacity. If she is able only to increase capacity, then she prefers to leave it unchanged since revenue decreases with capacity above the optimal level. The proofs of Lemma 1 and all subsequent proofs are in the Appendix.

Lemma 1. *The optimal commitment capacity $K^{\text{com}}(x)$ is decreasing in the quantity served x, and revenue $\rho^{\text{com}}(K, x)$ is strictly concave in K.*

Lemma 2 shows that as g and Δ shrink to zero, $\rho_{\Delta}^{\text{com}}(K, x, g)$ converges uniformly to $\rho^{\text{com}}(K, x)$.

Lemma 2. *Pick any $\underline{K} > 0$. If Assumption 1 holds, then for any $\varepsilon > 0$, there exist real numbers $c(\varepsilon) > 0$, $g(\varepsilon) > 0$, and $\Delta(\varepsilon) > 0$ such that whenever $c < c(\varepsilon)$, $g < g(\varepsilon)$, and $\Delta < \Delta(\varepsilon)$, then $|\rho_{\Delta}^{\text{com}}(K, x, g) - \rho^{\text{com}}(K, x)| < \varepsilon$ for all $K > \underline{K}$ and $x \in [0, q_0]$.*

To show uniform convergence, we argue that in the gap case, in equilibrium sales take place at rate K, as we assumed to be the case in deriving the properties of $\rho^{\text{com}}(K, x)$ in Lemma 1.[6] Thus, $\rho_{\Delta}^{\text{com}}(K, x, g)$ converges pointwise. Next, we obtain uniform convergence by bounding the effect on revenue of a marginal change in x or K. The marginal revenue effect of a change in the size of the market is no greater than the highest consumer valuation, which is an upper limit on willingness to pay. Raising capacity may either increase revenue, by allowing sales to be made more quickly, or decrease revenue, because buyers anticipating faster falls in prices are willing to pay less. We derive uniform bounds on both effects.

In Section 4, we return to augmentable capacity and apply the results of this section.

6. One way to see that result is to note that then

$$\rho^{\text{com}}(K, x) = \max_{0 \le dx/dt \le K} P(x)\frac{dx}{dt} + \rho^{\text{com}}(K, x) - r\rho^{\text{com}}(K, x) + \rho_x^{\text{com}}(K, x)\frac{dx}{dt}$$
$$= \frac{1}{r} \max_{0 \le dx/dt \le K} [P(x) + \rho_x^{\text{com}}(K, x)]\frac{dx}{dt}.$$

Since $\rho^{\text{com}}(K, x) > 0$, it must be that $[P(x) + \rho_x^{\text{com}}(K, x)] > 0$, and so the optimal rate of sales *dx/dt* is the upper bound, K.

© 2008 The Review of Economic Studies Limited

4. AUGMENTABLE CAPACITY

Now, we consider strictly positive gap g, capacity cost c, and period length Δ. We also allow the seller to increase capacity in any sales period. Our second main result is that in the limit, the monopolist earns commitment profits, even when she can augment capacity in every period.

Theorem 2. *If Assumption* 1 *holds, then for any $\varepsilon > 0$, there exist a real number $c(\varepsilon) > 0$ and integer-valued functions $g(c,\varepsilon) > 0$ and $\Delta(c,\varepsilon) > 0$ such that whenever $c < c(\varepsilon), g < g(c,\varepsilon)$, and $\Delta < \Delta(c,\varepsilon)$, then any subgame perfect equilibrium gives the monopolist a profit of at least* $\lim_{\Delta\to 0} \mathrm{P}_{\Delta}^{\mathrm{com}}(g) - \varepsilon$.

The intuition is as follows: suppose that play is one period away from a Coase path. That is, if $K\Delta$ units are sold in the current period, then next period's remaining market size will be such that in each period along the path of the Coase equilibrium for that market size sales do not exceed $K\Delta$; that is, the capacity constraint will not bind. In that case, any additional capacity that the monopolist purchases will be used in at most one sales period, so its marginal benefit is bounded by $v(0)\Delta$, which shrinks to zero as the period length Δ falls. The marginal cost c of capacity, on the other hand, does not vary with Δ, so for small enough Δ the monopolist will not increase capacity.

Now suppose that the firm has capacity K at least as great as K^{com}, and suppose that the size $q_0 - x$ of the remaining market is such that play is one sales period away from a subgame where the unique equilibrium entails never increasing capacity. In that case, this period's capacity will be the capacity for the rest of the game. The firm, then, would like to choose capacity $K^{\mathrm{com}}(x)$. However, Lemma 1 (plus an appropriate continuity argument following from Lemma 2) implies that existing capacity K is already greater than or equal to optimal capacity $K^{\mathrm{com}}(x)$, and that increasing capacity cannot raise profits. (The presence of a positive gap g tends to drive up the optimal capacity near the end of the market, but if g is small enough the new optimal capacity remains below the initial optimum. At the very end, so few consumers are left that even a very small cost is enough to deter an increase in capacity. That is, even if the revenue-maximizing capacity increases at the end of the market, the *profit*-maximizing change in capacity is zero.) Therefore, the firm will not purchase any additional capacity. Thus, by induction, if the monopolist chooses capacity K^{com} in the first period, she will never increase it, and so will earn the profits from committing to that capacity.

To summarize: Lemma 1 shows in the continuous time, no-gap case that i) optimal commitment capacity falls with x and ii) commitment profits fall in capacity above the optimal level. Thus, for $x > 0$, the new optimal capacity is strictly lower than the initial optimum (which the seller already has), and increasing capacity further will only lower revenue. In the discrete time, small gap case (with small enough period length and gap), the same will be true—that argument is Step 2 of the proof of Theorem 2. (Even if revenue is no longer quite decreasing in capacity, as may happen at the end of the market, a small cost of capacity ensures that profits are decreasing—that argument is Step 1.)

We showed in Section 3 that revenue in the commitment version of the game is at least a fraction $\gamma\,(\approx 0{\cdot}298)$ of static monopoly revenue $R(q_m)$. That result implies the following corollary of Theorem 2.

Corollary 1. *If Assumption* 1 *holds, then for any $\varepsilon > 0$, there exist a real number $c(\varepsilon) > 0$ and integer-valued functions $g(c,\varepsilon) > 0$ and $\Delta(c,\varepsilon) > 0$ such that if $c < c(\varepsilon), g < g(c,\varepsilon)$, and $\Delta < \Delta(c,\varepsilon)$, then any SPE gives the monopolist a profit of at least $\gamma\, R(q_m) - \varepsilon$.*

© 2008 The Review of Economic Studies Limited

Thus, rather than making the zero profit predicted by the Coase conjecture, the monopolist attains a substantial fraction of the profit that she could make if she could commit to a schedule of prices. Note that throughout our analysis, we take the limit as the capacity cost shrinks of the limit of profits given the cost as the time between sales periods shrinks. As discussed in the introduction, that is the appropriate order of limits: the cost is exogenous to the monopolist, while Coasian logic compels her to sell as quickly as possible.

5. CONCLUSION

This paper demonstrates that capacity costs of arbitrarily small degree can eliminate the zero profit conclusion of Ronald Coase's (1972) conjecture. Coasian dynamics—prices falling over time and quantities eventually exceeding the static monopoly quantity—prevail, but capacity choice provides a strong means of slowing the sales, thereby slowing the fall in prices, and thus permitting initial prices well in excess of marginal costs.

Whenever capacity is a choice, Coase's conjecture requires a monopolist to act in a manner not in her best interest. In order to implement the Coase path, the monopolist must invest in the resources to sell to all the buyers instantly. Usually, this investment will require some outlay; our result shows that even an arbitrarily small outlay serves as a strong commitment device for the monopolist. The monopolist cannot be compelled by the rational expectations of buyers to expand capacity beyond the profit-maximizing level. That is, buyers can expect rapid sales (and hence low prices) only if the monopolist creates the necessary capacity, but that investment is not in her best interest. When she fails to buy a large capacity, backwards induction forces the buyers to conclude that she will not expand capacity in the future, which makes the decision to choose low-capacity rational.

Thus, we find that the Coase conjecture is not robust to a very reasonable change in the specification of the environment. The logic of subgame perfection dictates that the monopolist continues to sell beyond the static monopoly level, but the ability to slow these sales by a smaller capacity choice, even in the limit when capacity becomes free, ensures that the monopolist earns a significant fraction of the static monopoly profits. The seller makes at least 29·8% of the monopoly profits, which is a far cry from zero.

Besides capacity costs, which apply in almost any setting, there are a variety of other means that in some situations may enable a durable-goods monopolist to escape the grim logic of the Coase conjecture. Leading the list is renting, which is mentioned in Coase's original article. A seller who rents, rather than sells, has no incentive to expand output beyond the monopoly quantity, for such an expansion entails a price cut not only to the new customers but also to existing customers. By allowing existing customers to renegotiate, rental serves a means of committing to a "most favoured customer" clause, in which early buyers are offered terms no worse than later buyers. Renting as a means of commitment has been offered as an explanation for IBM's rental of business machines (Wilson, 1993), although evidence is scant. Other solutions offered in the literature include return policies or money-back guarantees, destroying the production facilities, making the flow costs of staying in the market expensive (*e.g.* by renting the factory), concealing the marginal cost from buyers to interfere with their expectations about future prices, and planning obsolescence to eliminate the requisite perfect durability (see Tirole, 1988).

The analysis of Sobel (1991) suggests that entry of potential buyers (quite reasonable in light of finite human lifespans), which is isomorphic to imperfect durability, will create a price cycle. Prices tend to fall until it pays to sell to low-value consumers because sales to high-value consumers have made them relatively rare; once existing low-value consumers are satisfied, prices rise and sales are made only to newly born high-value buyers. This analysis was enhanced by Pesendorfer (1995) for goods with a network diseconomy. A logical conjecture is that the

© 2008 The Review of Economic Studies Limited

presence of capacity choice will enhance the seller's ability to dynamically price discriminate and lengthen the price cycle, but we have not investigated this formally. It is conceivable that the seller might choose such a low capacity so as to sell only to high-value buyers, thereby eliminating cycles altogether.

The Coase conjecture has been investigated in two finite versions. Bagnoli, Salant and Swierzbinski (1989) demonstrate that if there are finitely many potential buyers of known types, then there is a subgame perfect equilibrium with the seller extracting almost all the surplus. Essentially, she offers the good at a price near the maximum value; after that transaction, she offers at the next highest value, and so on. This model requires an unreasonable level of detailed knowledge on the part of the seller, but is interesting because it has such a different outcome from the continuous model. A bridge between the continuous and the discrete demand cases was developed by Levine and Pesendorfer (1995). von der Fehr and Kuhn (1995) show that if it is the seller who can set prices only in a discrete set, then her profits shrink to zero. The form of discreteness in our model (namely, that within a sales periods marginal cost is zero up to capacity and infinite above it) is qualititatively different, and it yields a very different outcome. McAfee and Vincent (1997) consider the Coasian auction problem, where the seller has one unit to sell to finitely many buyers who privately know their willingness to pay. They show that the opening reserve price exceeds marginal cost (even in the limit), but that the likelihood that the reserve price binds converges to zero as the periods come faster. Consequently, auction profits converge to the same level of profits arising from holding an auction with efficient reserve price. This model suggests a very different setting to consider capacity choice.

The Coase model can also be used to describe sequential bargaining between a seller with a single unit of a good and a buyer with a privately known valuation. The demand curve from the multiple-buyer case is then reinterpreted as the probability distribution of the single buyer's value. With capacity constraints, however, the two models are no longer equivalent. When there is only a single unit to be produced, capacity becomes unimportant.

Finally, we briefly examine the impact of allowing the monopolist the option to reduce as well as to augment capacity in each period. Suppose that in each sales period, the monopolist can either increase or decrease capacity, at a symmetric cost of c per unit. (For example, there may be a cost to closing down a factory or laying off workers.) In such an environment, the monopolist may be able to credibly commit to a sales path where the volume of sales per period decreases over time. Such a path could yield revenues even higher than the commitment profits in Section 2 because the monopolist sells to high-value consumers quickly. Those consumers are still willing to pay a high price because after they buy, prices will fall only very slowly as the monopolist reduces capacity.

APPENDIX. PROOFS

Proof that $-r\Delta/1-e^{-r\Delta}$ *is decreasing in* Δ *and* $\lim_{\Delta\to 0} -r\Delta/1-e^{-r\Delta} = -1$.

$$\frac{\partial}{\partial \Delta}\frac{-r\Delta}{1-e^{-r\Delta}} = \frac{-(1-e^{-r\Delta})+r\Delta e^{-r\Delta}}{(1-e^{-r\Delta})^2 r}$$

and thus $-r\Delta/1-e^{-r\Delta}$ is decreasing if $f(\Delta) = -(1-e^{-r\Delta})+r\Delta e^{-r\Delta} \leq 0$ for all $\Delta \geq 0$. But $f'(\Delta) = -r^2\Delta^2 e^{-r\Delta} \leq 0$, so $f(\Delta) \geq f(0) = 0$.

To show convergence to -1, note that

$$\lim_{\Delta\to 0}\frac{-r\Delta}{1-e^{-r\Delta}} = \lim_{\Delta\to 0}\frac{\partial(-r\Delta)/\partial\Delta}{\partial(1-e^{-r\Delta})/\partial\Delta}$$

$$= \lim_{\Delta\to 0}\frac{-r}{re^{-r\Delta}} = -1. \quad \|$$

© 2008 The Review of Economic Studies Limited

Proof of Lemma 1. Let $P(x,t)$ be the price charged at time t (starting from time 0 when quantity x has been sold). Following Stokey (1982), we note that the unique expression for the price path is

$$P(x,t) = e^{-r\left(\frac{q_0-x}{K}-t\right)}p(q_0) + re^{rt}\int_t^{\frac{q_0-x}{K}} e^{-rs}p(x+Ks)ds.$$

The seller's revenue is

$$\begin{aligned}
\rho^{\text{com}}(K,x) &= \int_0^{\frac{q_0-x}{K}} e^{-rt}KP(x,t)dt \\
&= \int_0^{\frac{q_0-x}{K}} e^{-rt}\left(Ke^{-r\left(\frac{q_0-x}{K}-t\right)}p(q_0) + re^{rt}\int_t^{\frac{q_0-x}{K}} e^{-rs}p(x+Ks)Kds\right)dt \\
&= \int_0^{\frac{q_0-x}{K}} \left(r\int_t^{\frac{q_0-x}{K}} e^{-rs}p(x+Ks)Kds\right)dt && \text{(using } p(q_0)=0\text{)} \\
&= rt\int_t^{\frac{q_0-x}{K}} e^{-rs}p(x+Ks)Kds\Big|_{t=0}^{t=\frac{q_0-x}{K}} + \int_0^{\frac{q_0-x}{K}} rte^{-rt}p(x+Kt)Kdt && \text{(integration by parts)} \\
&= \int_0^{\frac{q_0-x}{K}} rte^{-rt}p(x+Kt)Kdt \\
&= \int_x^{q_o} a(q-x)e^{-a(q-x)}p(q)dq && \text{(change of variables: } q=x+Kt \text{ and } a=r/K\text{)} \\
&= \int_x^{\infty} e^{-a(q-x)}a(q-x)p(q)dq && \text{(since } p(q)=0 \text{ for } q>q_0\text{)} \\
&= \int_0^{\infty} ze^{-z}p(x+z/a)\frac{dz}{a} && \text{(change of variables: } z=a(q-x)\text{)} \\
&= \int_0^{\infty} ze^{-z}p(x+bz)bdz && \text{(setting } b=1/a=K/r\text{).}
\end{aligned}$$

Therefore,

$$\frac{\partial\rho}{\partial b} = \int_0^{\infty} ze^{-z}(p(x+bz)+bzp'(x+bz))dz, \quad \text{and}$$

$$\begin{aligned}
\frac{\partial^2\rho}{\partial x\partial b} &= \int_0^{\infty} ze^{-z}(p'(x+bz)+bzp''(x+bz))dz \\
&= \int_x^{\infty} a(q-x)e^{-a(q-x)}(p'(q)+(q-x)p''(q))dq.
\end{aligned}$$

© 2008 The Review of Economic Studies Limited

Thus, a sufficient condition for $K^{\text{com}}(x)$ to decrease in x is $p' + qp'' \leq 0$. That this condition is sufficient follows from the following logic. If $p'' \leq 0$, then $p' + (q-x)p'' \leq 0$. If $p'' > 0$, then $p' + (q-x)p'' \leq p' + qp'' \leq 0$. Either way, the term is non-positive. Moreover,

$$\frac{\partial^2 \rho}{(\partial b)^2} = \int_0^\infty ze^{-z}(2p'(x+bz) + bzp''(x+bz))dz < 0,$$

so revenues are strictly concave in K, and the first order conditions uniquely characterize a maximum. ||

Proof of Lemma 2. First, we note that $\rho_\Delta^{\text{com}}(K,x,g)$ converges to $\rho^{\text{com}}(K,x)$ pointwise. (If we introduce any positive gap g in the continuous time, commitment case, then the monopolist will sell to the whole market in finite time, given $K > 0$, and there is a unique backwards induction solution. Coasian dynamics imply that in that equilibrium, at each instant sales are made up to capacity; that is, at rate K.) Similarly, for high levels of capacity K, all sales will take place almost instantly, and revenue will be close to $g(q_0 - x)$. There exists a $\bar{K}$, then, such that when $K > \bar{K}$, both $\rho_\Delta^{\text{com}}(K,x,g)$ and $\rho^{\text{com}}(K,x)$ converge uniformly to zero as g and Δ shrink.

It remains only to show uniform convergence at $K \in [\underline{K}, \bar{K}]$. We first bound the marginal effect on $\rho_\Delta^{\text{com}}(K,x,g)$ and $\rho^{\text{com}}(K,x)$ of small changes in x and K. The marginal increase in revenue from increasing the size of the remaining market is no greater than $\upsilon(0)(=p(0)+g)$. Thus, if $|x - x'| < \varepsilon/5p(0)$, then $|\rho\Delta^{\text{com}}(K,x,g) - \rho_\Delta^{\text{com}}(K,x',g)|$ and $|\rho^{\text{com}}(K,x) - \rho^{\text{com}}(K,x')|$ are both less than $\varepsilon/5$ for small enough g.

An increase in capacity may increase commitment revenues by allowing sales to be made more quickly. If the capacity constraint binds in every period, then the waiting time until the last buyer is served is q_0/K, rounded up to the nearest Δ. (Call that value T.) Thus, an increase in K that reduces the value of q_0/K by d might decrease that waiting time by up to $d+\Delta$. The resulting increase in the discounted value of revenue, then, is no more than

$$e^{-r(T-(d+\Delta))}\upsilon(0)q_0 - e^{-rT}\upsilon(0)q_0 \leq (e^{r(d+\Delta)} - 1)\upsilon(0)q_0, \tag{A1}$$

which bounds the increase in case i) all sales take place in the last period, ii) all buyers pay $\upsilon(0)$, and iii) the monopolist does not have to charge lower prices in order to sell more quickly.

Conversely, raising capacity can reduce revenue if buyers expect faster falls in prices. Since consumer q is indifferent between waiting $d+\Delta$ units of time to pay the last price g and paying $e^{-r(d+\Delta)}g + (1-e^{-r(d+\Delta)})\upsilon(q)$ now, the reduction in revenue that results from lowering the value of q_0/K by d (and thus decreasing the waiting time by up to $d+\Delta$) is no greater than

$$\begin{aligned}&[e^{-rT}g + (1-e^{-rT})\upsilon(0)]q_0 - [e^{-r(T-(d+\Delta))}g + (1-e^{-r(T-(d+\Delta))})\upsilon(0)]q_0 \\ &\leq (e^{r(d+\Delta)} - 1)[\upsilon(0) - g]q_0.\end{aligned} \tag{A2}$$

That bound applies even if all consumers have the highest valuation $\upsilon(0)$, and the reduction in revenue is not discounted. Because $g > 0$, the magnitude of expression (A2) is strictly less than that of expression (A1).

Choose a d^* satisfying $\left(e^{rd^*} - 1\right)\upsilon(0)q_0 < \varepsilon/5$. If $|K - K'| < (\underline{K}^2/q_0)d^*$, then

$$|q_0/K - q_0/K'| < d^*.$$

Thus, if Δ is small enough and $|K - K'| < (\underline{K}^2/q_0)d^*$, then the values of the distances $|\rho_\Delta^{\text{com}}(K,x,g) - \rho_\Delta^{\text{com}}(K',x,g)|$ and $|\rho^{\text{com}}(K,x) - \rho^{\text{com}}(K',x)|$ are both less than $\varepsilon/5$ for all $K, K' \in [\underline{K}, \bar{K}]$.

Finally, choose a finite subset $F \subseteq [\underline{K}, \bar{K}] \times [0, q_0]$ such that every point in $[\underline{K}, \bar{K}] \times [0, q_0]$ is within $\min\{\varepsilon/5p(0), (\underline{K}^2/q_0)d^*\}$ of an element of F. Since $\rho_\Delta^{\text{com}}(K,x,g)$ converges to $\rho^{\text{com}}(K,x)$ pointwise, we can choose Δ and g small enough that, in addition to the conditions above, $|\rho_\Delta^{\text{com}}(K,x,g) - \rho^{\text{com}}(K,x)| < \varepsilon/5$ for every $(K,x) \in F$. Now, pick any point $(K,x) \in [\underline{K}, \bar{K}] \times [0, q_0]$, and let (K_F, x_F) be the nearest element of F. By construction, $|\rho_\Delta^{\text{com}}(K,x,g) - \rho^{\text{com}}(K,x)| < \varepsilon$. ($\rho_\Delta^{\text{com}}(K,x,g)$ is within $\varepsilon/5$ of $\rho_\Delta^{\text{com}}(K_F,x,g)$, which is within $\varepsilon/5$ of $\rho_\Delta^{\text{com}}(K_F,x_F,g)$, which is within $\varepsilon/5$ of $\rho^{\text{com}}(K_F,x_F)$, which is within $\varepsilon/5$ of $\rho^{\text{com}}(K_F,x)$, which is within $\varepsilon/5$ of $\rho^{\text{com}}(K,x)$.) ||

Proof of Theorem 2. As a preliminary, we introduce four new definitions. Let the function $S^{\text{C}}(x,g,\Delta)$ denote the quantity sold in the first period of the Coase equilibrium (*i.e.* without capacity constraints) with market size $q_0 - x$, gap g, and period length Δ. Define $S_{\max}^{\text{C}}(x,g,\Delta)$ as the maximum quantity sold in any period along that Coase equilibrium path. The quantity sold in the first period of the equilibrium when capacity is fixed at K, the served market is x, the gap is g, and the period length is Δ is given by $S_\Delta^{\text{com}}(K,x,g)$. Lastly, let $K_\Delta^{\max}(g,c)$ be the highest optimal commitment

© 2008 The Review of Economic Studies Limited

capacity over all possible market sizes. That is,

$$K_{\Delta}^{\max}(g,c) = \max_{x\in[0,q_0]}\left(\arg\max_{K>0} \rho_{\Delta}^{\text{com}}(K,x,g) - cK\right).$$

The proof is inductive. First, we say that play is one sales period away from a Coase path if i) current capacity is less than the Coase sales quantity and ii) the market that remains after the monopolist sells her capacity in the current period is such that the quantities sold in each period in that market's Coase equilibrium are no greater than current capacity—that is, if

$$S^{\text{C}}(x_z, g, \Delta) > K_{z-1}\Delta \geq S_{\max}^{\text{C}}(x_z + K_{z-1}\Delta, g, \Delta).$$

We will show that if play is one sales period away from a Coase path, then there is a unique subgame perfect continuation, in which the monopolist will never increase capacity. Note that the capacity constraint cannot bind in the last period of sales, and the monopolist will eventually sell to the entire market, so eventually play must be on a Coase path. Second, we say that play is one sales period away from a unique continuation with constant capacity if for all market sizes $x \geq x_z + S_{\Delta}^{\text{com}}(K_{z-1}, x_z, g)$ and all $K \geq K_{z-1}$, there is a unique SPE, and in that SPE capacity is never increased. We will show that if $K_{z-1} \geq K_{\Delta}^{\max}(g,c)$, and play is one sales period away from a unique continuation with constant capacity, again there is a unique subgame perfect continuation, in which the monopolist will never increase capacity. Next, we show that by choosing initial capacity equal to $K_{\Delta}^{\max}(g,c)$, the monopolist can guarantee profits of at least $\rho_{\Delta}^{\text{com}}(K_{\Delta}^{\max}(g,c),g) - cK_{\Delta}^{\max}(g,c)$. That observation is sufficient to establish the result, since Lemmas 1 and 2 ensure that $\rho_{\Delta}^{\text{com}}(K_{\Delta}^{\max}(g,c),g)$ converges to $\lim_{\Delta\to 0} \text{P}_{\Delta}^{\text{com}}(g)$. (In Lemma 2, take $\underline{K}$ as $K^{\text{com}}(0)/2$. Since Lemma 1 implies that $K^{\text{com}}(0)$ is the unique maximizer of $\rho^{\text{com}}(K,0)$, that $\rho^{\text{com}}(K,x)$ is strictly concave in K, and that $K^{\text{com}}(x)$ is decreasing, the uniform convergence of $\rho_{\Delta}^{\text{com}}(K,x,g)$ means that $K_{\Delta}^{\max}(g,c)$ converges to $K^{\text{com}}(0)$, and thus that $\rho_{\Delta}^{\text{com}}(K_{\Delta}^{\max}(g,c),g)$ converges to $\lim_{\Delta\to 0} \text{P}_{\Delta}^{\text{com}}(g)$.)

Step 1. One sales period from the Coase path.

First, note that the monopolist, having chosen $K_z \geq K_{z-1}$, will set a price so as to sell either $K_z\Delta$ or the Coase quantity $S^{\text{C}}(x_z, g, \Delta)$, if $K_z\Delta$ exceeds $S^{\text{C}}(x_z, g, \Delta)$. Let x^* equal $x_z + \min\{K_z\Delta, S^{\text{C}}(x_z, g, \Delta)\}$. Fudenberg *et al.* (1985) show that the optimal action (subject to subgame perfection) for the monopolist when i) x lies in the interval $[x^* - S^{\text{C}}(x_z, g, \Delta), x^*]$ and ii) the continuation from x^* is on the Coase path, is to sell volume $x^* - x_z$ at the Coase price. Thus, the quantity sold S in period z is equal to the smaller of $K_z\Delta$ and $S^{\text{C}}(x_z, g, \Delta)$.

Whether or not the monopolist increases capacity above K_{z-1}, then, play starting next period will be on a Coase path, where the capacity constraint never binds. Therefore, any additional capacity the monopolist purchases in period z will be used at most once. Furthermore, because Coase profits are decreasing in x, and $S = \min\{K_z\Delta, S^{\text{C}}(x, g, \Delta)\}$, continuation profits are no greater than the level that results from not increasing capacity. By similar reasoning, the price charged in period z is no higher than the price if capacity is not increased. Thus, the marginal revenue of increasing capacity in period z is bounded above by the highest consumer valuation, $\upsilon(0)$, times the amount of the additional capacity used in one period, Δ. That bound, $\upsilon(0)\Delta$, shrinks to zero as Δ falls. The marginal cost of raising capacity, on the other hand, is the constant c. Thus, if Δ is small enough, the only subgame perfect action for the monopolist is to choose $K_z = K_{z-1}$.

Step 2. One sales period from a unique continuation with constant capacity and $K_{z1} \geq K_{\Delta}^{\max}(g,c)$.

First, note that since $K_z \geq K_{z-1} \geq K_{\Delta}^{\max}(g,c) \geq S_{\Delta}^{\text{com}}(K_{z-1}, x_z, g)$, if the monopolist does not increase capacity in period z, she will set a price so as to sell quantity $S_{\Delta}^{\text{com}}(K_{z-1}, x_z, g)$, by definition of $S_{\Delta}^{\text{com}}(K_{z-1}, x_z, g)$. Note also that the monopolist's maximal revenue consistent with subgame perfection if she does increase capacity to $K_z > K_{z-1}$ is bounded above by $\rho_{\Delta}^{\text{com}}(K_z, x_z, g)$, the profit if the monopolist can commit to never again increasing capacity. (The only way that the maximal revenue without commitment could be higher than the commitment revenue is if the monopolist would optimally increase capacity after the first period. In the present case, however, the induction hypothesis ensures that capacity will not be increased when the served market x rises to $x_z + S_{\Delta}^{\text{com}}(K_{z-1}, x_z, g)$ or higher, and the monopolist already has sufficient capacity to reach that level in one's sales period.) Since i) current capacity K_{z-1} already weakly exceeds $K_{\Delta}^{\max}(g,c)$, ii) Lemma 1 implies that $\rho^{\text{com}}(K, x_z)$ is decreasing in K above that level, and iii) Lemma 2 shows that $\rho_{\Delta}^{\text{com}}(K, x_z, g)$ converges uniformly to $\rho^{\text{com}}(K, x_z)$, for small enough g and Δ the value of $\rho_{\Delta}^{\text{com}}(K_z, x_z, g) - c(K_z - K_{z-1})$ cannot increase with $K_z \geq K_{z-1}$. In order to maximize profits, then, the monopolist will not increase capacity when play is one period away from a unique continuation with constant capacity and $K_{z-1} \geq K_{\Delta}^{\max}(g,c)$.

© 2008 The Review of Economic Studies Limited

That ends the induction. Thus, the monopolist can guarantee herself a profit of $\rho_{\Delta}^{\text{com}}(K_{\Delta}^{\max}(g,c),g) - cK_{\Delta}^{\max}(g,c)$ by purchasing capacity $K_{\Delta}^{\max}(g,c)$ in the first period. Any SPE, therefore, must give the monopolist at least that level of profits. ‖

Acknowledgements. We thank Ken Hendricks and Philip Reny for their stubborn unwillingness to be convinced by incorrect arguments. We also thank seminar audiences and Angela Young for helpful comments and anonymous referees for detailed suggestions.

REFERENCES

AUSUBEL, L. and DENECKERE, R. (1989), "Reputation in Bargaining and Durable Goods Monopoly", *Econometrica*, **57**, 511–532.

BAGNOLI, M., SALANT, S. W. and SWIERZBINSKI, J. E. (1989), "Durable-Goods Monopoly with Discrete Demand", *Journal of Political Economy*, **97**, 1459–1478.

BULOW, J. (1982), "Durable Goods Monopolists", *Journal of Political Economy*, **90**, 314–332.

COASE, R. (1972), "Durability and Monopoly", *Journal of Law and Economics*, **15**, 143–149.

DENICOLO, V. and GARELLA, P. G. (1999), "Rationing in a Durable Goods Monopoly", *RAND Journal of Economics*, **30**, 44–55.

FUDENBERG, D., LEVINE, D. and TIROLE, J. (1985), "Infinite Horizon Models of Bargaining with One-Sided Incomplete Information", in A. Roth (ed.) *Game Theoretic Models of Bargaining* (Cambridge: Cambridge University Press).

GUL, F., SONNENSCHEIN, H. and WILSON, R. (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture", *Journal of Economic Theory*, **39**, 155–190.

KAHN, C. (1986), "The Durable Goods Monopoly and Consistency with Increasing Costs", *Econometrica*, **54**, 275–294.

LEVINE, D. and PESENDORFER, W. (1995), "When are Agents Negligible?", *American Economic Review*, **85**, 1160–1170.

MCAFEE, R. P. and VINCENT, D. (1997), "Sequentially Optimal Auctions", *Games and Economic Behavior*, **18**, 246–276.

PESENDORFER, W. (1995), "Design Innovation and Fashion Cycles", *American Economic Review*, **85**, 771–792.

SOBEL, J. (1991), "Durable Goods Monopoly with Entry of New Consumers", *Econometrica*, **59**, 1455–1485.

STOKEY, N. (1979), "Intertemporal Price Discrimination", *Quarterly Journal of Economics*, **93**, 355–371.

STOKEY, N. (1982), "Rational Expectations and Durable Goods Pricing", *Bell Journal of Economics*, **12**, 112–128.

TIROLE, J. (1988) *The Theory of Industrial Organization* (Cambridge, MA: MIT Press).

VON DER FEHR, N.-H. M. and KUHN, K. U. (1995), "Coase versus Pacman: Who Eats Whom in the Durable-Goods Monopoly?", *Journal of Political Economy*, **103**, 785–812.

WILSON, R. B. (1993) *Nonlinear Pricing* (Oxford: Oxford University Press).

© 2008 The Review of Economic Studies Limited