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AUCTIONS WITH ENTRY

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We consider an auction which bidders can enter upon paying an entry cost. Unlike the case of a fixed number of bidders, the seller should not impose a reserve price higher than his own valuation. When a first-price sealed-bid auction is used, the optimal number of bidders enter.

1. Introduction

A robust prediction of auction theory is that reserve prices will be used in auctions. The optimal selling mechanism for the owner of a unique, indivisible item is to sell to the bidder with the highest valuation unless this valuation is below a cut-off level; and this cut-off valuation is strictly above the seller's own valuation. Thus the monopolist imposes a distortion: with positive probability he will refuse to sell the item even though there is a bidder who values it more than he does. [1]

In practice, reserve prices are often, perhaps usually, not used in auctions [Cassady (1967, pp. 226–227)]. Thus, there appears to be a discrepancy between theory and practice.

In this paper, we relax the assumption, universally used in auction theory, that there is a fixed number of bidders. Instead, we make the assumption, arguably more economically reasonable, that any bidder can enter the bidding upon paying an entry cost (which may be interpreted as the cost of learning what the item is worth or the cost of preparing a bid). We find that the seller should never reject a bidder with a valuation higher than his own. In addition we find that, if the seller uses a first-price sealed-bid auction, the optimal number of bidders will enter. Thus, the optimal auction in this case is the first-price sealed-bid auction with a reserve price equal to the seller's own valuation. The seller designs an efficient selling mechanism. If we add the non-restrictive assumption that any bidder's valuation exceeds the seller's, then, consistent with observation, no reserve price is used. [2]

In the usual auction analysis, with the number of bidders given exogenously, the seller uses the reserve price to extract rents from the winning bidder. But with entry of bidders, the winner's rent is fixed, regardless of the seller's policy. Thus there is no gain from introducing a distortion by using a reserve price.

[1] Laffont and Maskin (1980), Harris and Raviv (1981), Myerson (1981), Riley and Samuelson (1981), Milgrom and Weber (1982), and McAfee and McMillan (1987b). For surveys of auction theory, see Milgrom (1986) and McAfee and McMillan (1987a).

[2] Milgrom (1986) offered another reason for the seller's not imposing a reserve price: his inability to precommit himself to his selling mechanism.

Although the analysis will be expressed in terms of a monopolist designing a selling mechanism, it is easily converted to the case of a single buyer accepting bids from potential suppliers: in particular, the model would then apply to government procurement. The United States Department of Defense has a dual-sourcing policy, which involves subsidizing an extra firm's costs of investment specific to the production of a particular weapons system in order to generate bidding competition against the incumbent producer. If the assumptions of the present model are appropriate, such interventions are not warranted: if the number of firms currently capable of doing the work is too small, it is in an entrant's own interest to incur the costs of entry. The laissez-faire number of firms is optimal. There should be a single supplier if the cost of setting up production facilities is large enough relative to the potential dispersion of production costs across bidding firms.

2. Entry of bidders

We assume that there are arbitrarily many potential bidders, any of whom can enter the bidding upon paying a fixed cost $k \geq 0$. After paying this entry cost, the bidder learns his valuation of the item for sale, v. The remaining assumptions are standard [as in Myerson (1981) and Riley and Samuelson (1981)]. The bidders' valuations are drawn independently from a distribution F with density f on a support $[v_l, v_h]$. We assume the standard regularity condition on the distribution F, namely, the expression $[v-(1-F(v))/f(v)]$ is monotonically increasing in v. This ensures that the seller does not choose a randomized strategy [Myerson (1981)]. The seller and the bidders are assumed to be risk neutral.

We show first that, with entry of bidders, the seller should always offer the item to the highest-valuation bidder: he should never reject a bidder with valuation higher than his own. The reserve price is his own valuation.

This result is easily established. Bidders will enter until their expected profits are driven down to equal the entry cost k. [3] (We are temporarily ignoring the fact that there must be an integer number of bidders, so that profit only approximately equals k.) The seller's expected revenue is the winning bidder's expected valuation $\mathrm{E}v$ minus the expected profit of the n bidders that enter, or $\mathrm{E}v - nk$. Thus, for any given number of bidders n, the seller should award the good so as to maximize expected valuation $\mathrm{E}v$. This he does by setting a trivial reserve price, equal to his own valuation.

It remains to establish the number of bidders n. We shall show that the seller's using a first-price sealed-bid auction induces the optimal number of bidders to enter.

In order to show the optimality of the sealed-bid auction, we appeal to the Revelation Principle and optimize over the class of direct mechanisms. Suppose, then, that the seller asks each of the n bidders to report his or her valuations. Denote by w the report of one of the bidders. The seller asks for a payment of $p(w)$ from the bidder with the highest reported valuation. (Since the bidders are risk neutral and valuations are independent, nothing is gained by either requiring payment from or subsidizing the losing bidders [Myerson (1981), Riley and Samuelson (1981)].) If the remaining $n-1$ bidders report honestly, then this bidder wins with probability $[F(w)]^{n-1}$. His expected profit is, therefore,

$$\pi(w, v) = [v - p(w)][F(w)]^{n-1}. \tag{1}$$

[3] French and McCormick (1984) noted that, with the equilibrium number of bidders, the winner's expected profit equals the sum of the bidders' entry costs.

By the Envelope Theorem, at an incentive-compatible Nash equilibrium

$$\frac{\mathrm{d}}{\mathrm{d}v}\pi(w, v) = \frac{\partial}{\partial v}\pi(w, v)\bigg|_{v=w} = [F(v)]^{n-1}. \tag{2}$$

Thus the equilibrium expected profit is

$$\pi(v, v) = \pi_0 + \int_{v_1}^{v}[F(\xi)]^{n-1}\,\mathrm{d}\xi, \tag{3}$$

where $\pi_0 = \pi(v_1, v_1)$ is the expected profit of a bidder with the lowest possible valuation.

The ex ante expected profit (before he draws his valuation) of any bidder is

$$\hat{\pi} = \int_{v_1}^{v_h}\pi(\xi, \xi)f(\xi)\,\mathrm{d}\xi = \pi_0 + \int_{v_1}^{v_h}\int_{v_1}^{v}[F(\xi)]^{n-1}\,\mathrm{d}\xi f(v)\,\mathrm{d}v$$

$$= \pi_0 + \int_{v_1}^{v_h}[F(v)]^{n-1}[1 - F(v)]\,\mathrm{d}v. \tag{4}$$

(The last line is obtained by integration by parts.)

The possibility of entry constrains the bidders' profits. Still allowing the number of bidders to be non-integer, bidders will enter until the expected profit (4) equals the entry cost k:

$$k = \pi_0 + \int_{v_1}^{v_h}[F(v)]^{n-1}[1 - F(v)]\,\mathrm{d}v. \tag{5}$$

This determines the number of bidders n. Note that

$$\frac{\mathrm{d}n}{\mathrm{d}\pi_0} = -1\bigg/\left(\frac{\partial}{\partial n}\int_{v_1}^{v_h}[1 - F(v)]\,\exp[(n-1)\log F(v)]\,\mathrm{d}v\right)$$

$$= -\left[\int_{v_1}^{v_h}[1 - F(v)]\,\log F(v)[F(v)]^{n-1}\,\mathrm{d}v\right]^{-1} > 0. \tag{6}$$

That is, by increasing π_0, the profit guaranteed to a bidder with the lowest possible valuation, the seller is able to increase the number of competing bidders.

What is the optimal number of bidders? As noted, the seller's expected revenue R is the winning bidder's expected valuation minus the total expected profit of all the bidders:

$$R = \int_{v_1}^{v_h} vn[F(v)]^{n-1}f(v)\,\mathrm{d}v - nk = v_h[F(v_h)]^n - \int_{v_1}^{v_h}[(F(v)]^n\,\mathrm{d}v - nk \tag{7}$$

(by integrating by parts). Hence

$$\frac{\partial R}{\partial n} = -\int_{v_1}^{v_h}\log F(v)[F(v)]^n\,\mathrm{d}v - k, \tag{8}$$

and

$$\frac{\partial^2 R}{\partial n^2} = -\int_{v_1}^{v_h}\log\,[F(v)]^2[F(v)]^n\,\mathrm{d}v < 0. \tag{9}$$

Thus, R is strictly concave in n, so that the optimal number of bidders n^* is found by setting eq. (8) equal to zero. Together with (5), this implies

$$\pi_0 = -\int_{v_l}^{v_h} [F(v)]^{n^*-1}[1 - F(v) + F(v) \log F(v)] \, dv, \tag{10}$$

which implies $\pi_0 < 0$. [To see this, set $x = F(v)$. Note that $1 - x + x \log x = 0$ when $x = 1$. Also $d(1 - x + x \log x)/dx = \log x < 0$ for $0 < x < 1$. Thus $1 - x + x \log x$ is decreasing and equals zero at $x = 1$, which implies $1 - x + \log x > 0$ for $0 < x < 1$.]

Hence the optimal number of bidders enter when the seller sets π_0, the profit of a bidder with the lowest possible valuation, negative: bidders with low valuations will on average lose upon bidding. The bidders' individual rationality means that the seller must extract these payments of π_0 before any bidder learns his valuation: this could be done, for example, by demanding a fee before potential bidders inspect the item for sale. If this payment were extracted after the bidders learned their valuations, then bidders with low valuations would not find it in their interest to pay π_0 and bid. [4]

Differentiation of (7) and (10) yields the following comparative-statics results on changes in the entry cost k. As k increases, the seller's expected revenue decreases, the optimal number of bidders decreases, and the minimum payment π_0 increases. In the limit, as the entry cost approaches zero, the equilibrium number of bidders approaches infinity (the market becomes perfectly competitive) and the minimum payment π_0 approaches zero.

How does the outcome of a first-price sealed-bid auction compare with this optimal mechanism? In a sealed-bid auction, $\pi_0 = 0$: no bidder can be forced to bid so as to make a loss. Thus, still allowing the number of bidders to be non-integer, the number of bidders choosing to enter under a sealed-bid auction, n_s, is defined by

$$k = \int_{v_l}^{v_h} [F(v)]^{n_s-1}[1 - F(v)] \, dv. \tag{11}$$

From (7),

$$R(n_s) - R(n_s - 1) = -k + \int_{v_l}^{v_h} [F(v)]^{n_s-1}[1 - F(v)] \, dv, \tag{12}$$

which, by (11), is equal to zero. R was previously shown to be strictly concave in n. Thus it must be that $n_s - 1 \leq n^* \leq n_s$.

Consider now the implications of the need for the number of bidders n to be an integer. If the bidders use a pure strategy, then the number of bidders entering the sealed-bid auction will equal the largest integer less than or equal to n_s. But, from the above argument, this is the closest integer to n^*. Among the integers, n_s is optimal: from the point of view of the seller, exactly the right number of bidders enter the sealed-bid auction. Thus we have shown that the well-known result that the first-price sealed-bid auction is an optimal mechanism [Harris and Raviv (1981), Myerson (1981), Riley and Samuelson (1981)] extends to the case where new bidders can enter; however, entry means that the reserve price should be the seller's own valuation.

To the extent that the number of bidders is less than n_s because of the integer restriction, the bidders in the sealed-bid auction earn positive rents (they would be zero if n_s were itself an integer).

[4] Samuelson (1985) analyzed a bidding model in which each bidder incurs a cost before bidding. Samuelson's model differs from the present model in that the bidder pays his fixed cost only after observing his valuation (so that a bidder with a low enough valuation will not bid), and in having a fixed number of potential bidders.

If the seller were able to charge a bidding fee before the bidders learn their valuations, he could extract this rent. Otherwise, the integer restriction leaves the bidders with positive surplus on average.

Finally, if the cost of entry is large enough, the optimal number of bidders may be one; that is, bilateral monopoly is optimal for the seller. This is the case if [from (8) and (9)]

$$-\int_{v_l}^{v_h} \log F(v)[F(v)]\ \mathrm{d}v - k \leq 0. \tag{13}$$

A sufficient condition for this is

$$k \geq (v_h - v_l)/e \tag{14}$$

(since $\log F(v)[F(v)] \geq -e^{-1}$); that is, the entry cost is sufficiently large relative to the possible spread in valuations. Returning to the government-procurement application discussed in the introduction, this is a sufficient condition for sole-source procurement to minimize the government's expected procurement cost.

3. Conclusion

In an auction which bidders can enter upon paying an entry cost, the optimal reserve price is the seller's own valuation, and a first-price sealed-bid auction induces the optimal number of bidders to enter.

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