Price Dispersion

> One should hardly have to tell academicians that information is a valuable resource: knowledge *is* power. And yet it occupies a slum dwelling in the town of economics.
> -- George Stigler's 1961 "The Economics of Information"

Diamond Paradox.

- Suppose that there is global minimim to search costs γ, that is, all consumers have search costs in excess of $\gamma>0$.
- Search costs arise per store – each store sampled costs γ or more.
- Unique equilibrium: all firms charge the monopoly price.

The Butters Model

Suppose n firms send advertisements to a proportion α of the population with price offers.

Each consumer chooses the price offer that is lowest.

Let $F(p)$ be the probability that a price offer is not more than p.

A firm's profits per consumer are

$$\pi(p) = (p-c)[1-\alpha + \alpha(1-F(p))]^{n-1}$$

Thus

$$\pi(p) = \pi(v) = (v-c)[1-\alpha]^{n-1}$$

Therefore

$$1-\alpha F(p) = 1-\alpha+\alpha(1-F(p)) = (1-\alpha)\left(\frac{v-c}{p-c}\right)^{\frac{1}{n-1}}.$$

or

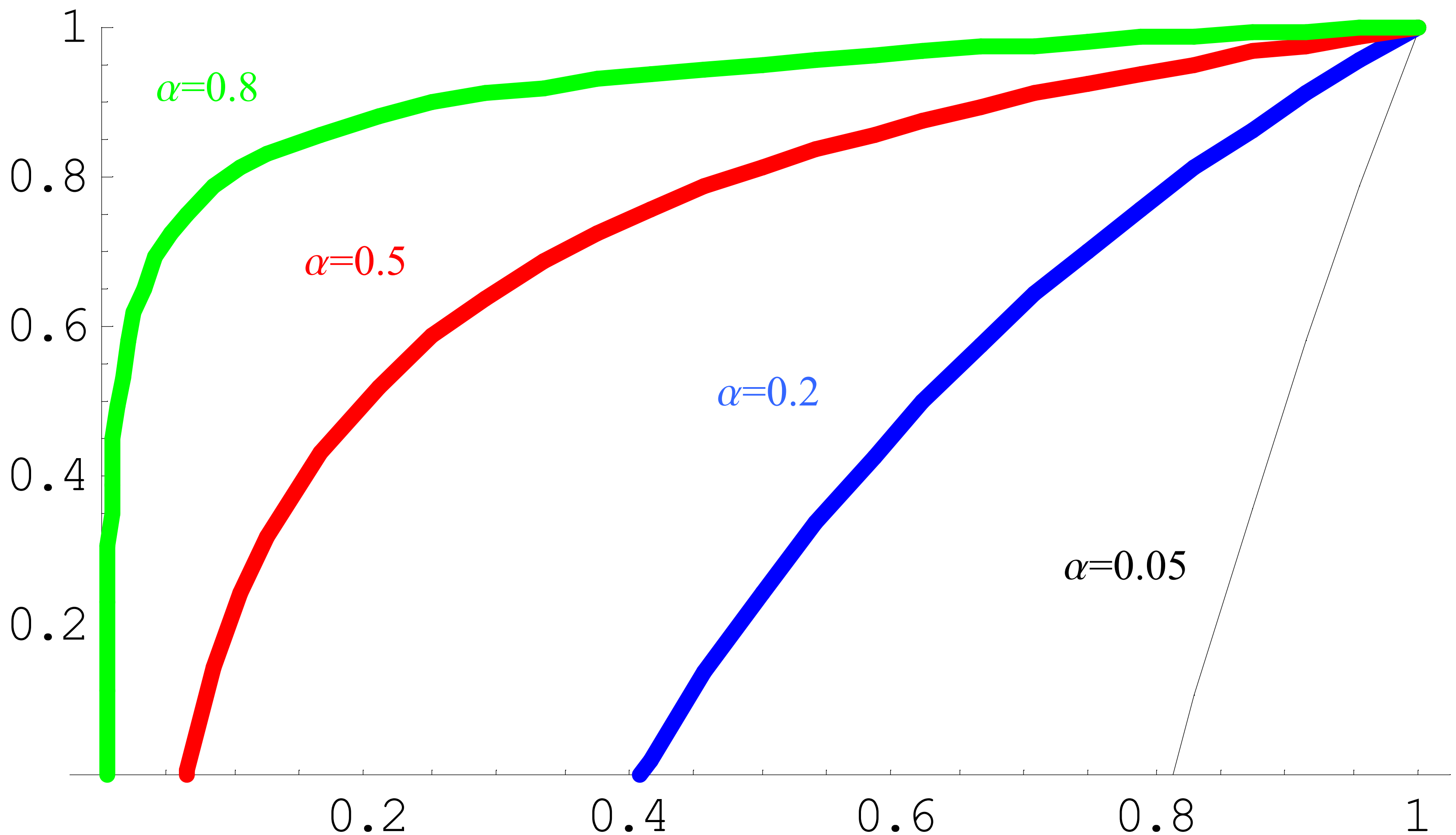
$$F(p) = \frac{1}{\alpha}\left[1-(1-\alpha)\left(\frac{v\text{ - }c}{p\text{ - }c}\right)^{\frac{1}{n-1}}\right].$$

The lowest price, p_o, is the price such that $F(p_o)$=0, or $p_o = c + (1\text{-}\alpha)^{n\text{-}1}(v\text{-}c)$.

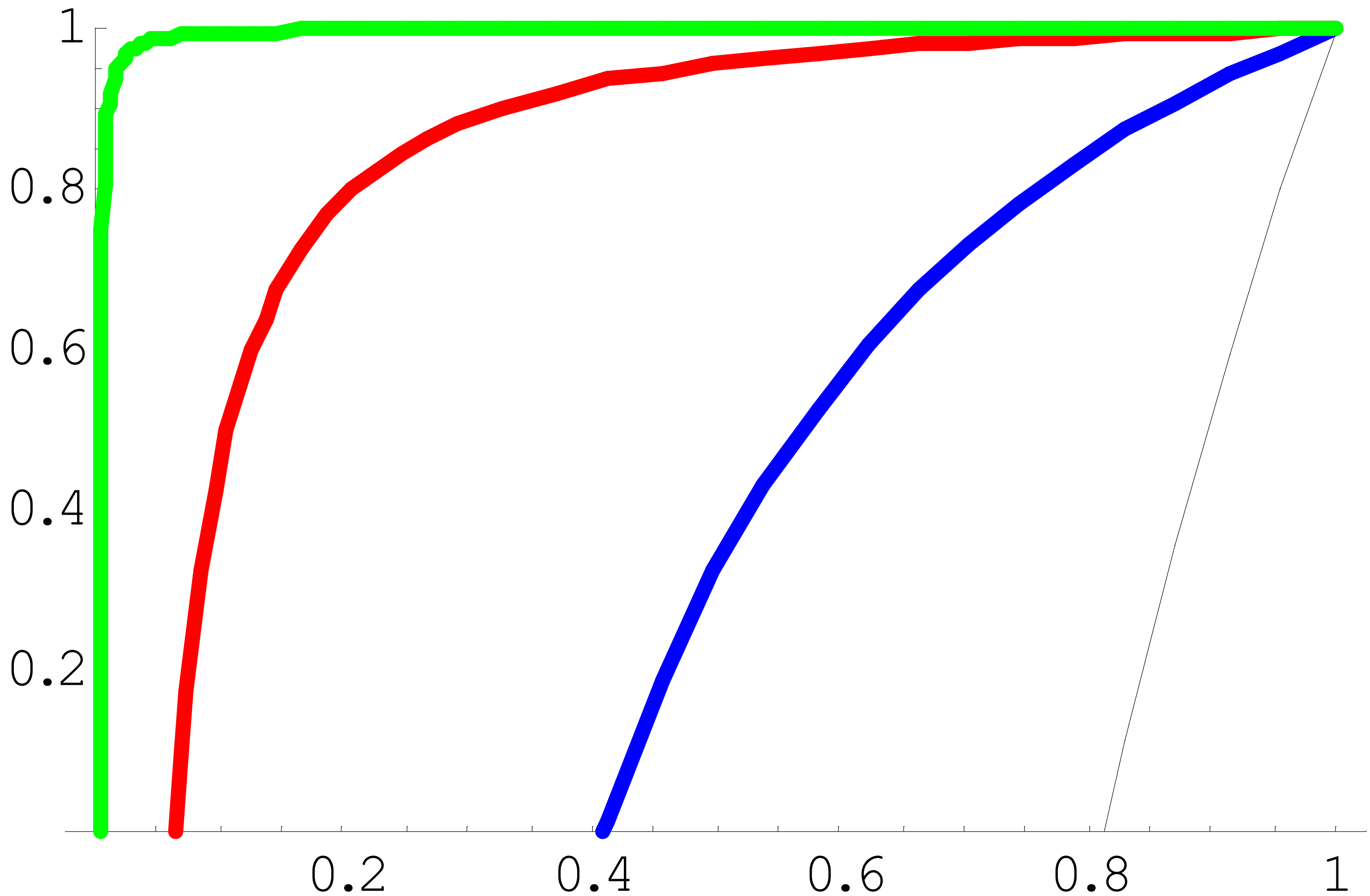
The distribution of prices is Prob(offer$\leq p$) = 1 - $[1\text{-}\alpha + \alpha(1\text{-}F(p)]^{n\text{-}1}$.

Thus the expected price minus cost is

$$E[p-c] = \int_{p_0}^{v}(p-c)n(1-\alpha F(p))^{n-1}\alpha f(p)dp = \int_{p_0}^{v}(p-c)n\pi\alpha f(p)dp = n\pi\alpha = n\alpha(v-c)(1-\alpha)^{n-1}.$$



The probability distribution of offered prices for various values of α, and the best price realized by consumers. c=0 and n=5.


1
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1

The solution to the Butters model is equivalent to the auction model in which bidders have independently distributed values in {0,1}, and α is the probability that a given bidder has value equal to 1. Such a bidder uses a mixed strategy F.

The Butters model can be varied in a straightforward way by letting buyers either know one price (chosen randomly) or all the prices, which is accomplished in Varian's AER paper. Adding a quantity choice that depends on price introduces no complexity. This gives a profit function

$$\pi(p) = (p-c)q(p)\left(\beta/n + (1-\beta)(1-F(p))^{n-1} \right)$$

Asymmetric Price Dispersion

Firms have availability rate α_i, ranked from largest to smallest, so that $\alpha_1 \geq \alpha_2 \geq \ldots$

Let $R(p)=(p-mc)q(p)$ and p^m maximize R.

Then with a substantial amount of work, one can show that there is an equilibrium, that firm 1 has a mass point at p^m if $\alpha_1 > \alpha_2$, that the firms with lower availability randomize over intervals with lower prices, and finally that profits per unit of availability are the same, but that the largest firm enters asymmetrically into the profit equation.

$$\pi_i = \alpha_i R(p^m) \prod_{j \neq 1} (1 - \alpha_j).$$

Introduce a cost $c(\alpha)$ of availability.

Given the multiplicative nature of probabilities, the size of a scale economy is given by the cost saving associated with combining the operations of two entities, which is

$c(\alpha)+c(\beta)-c(1-(1-\alpha)(1-\beta))$.

Consequently, availability has increasing returns to scale whenever

$$c(\alpha)+c(\beta)-c(1-(1-\alpha)(1-\beta)) \text{ is increasing in } \alpha \text{ or } \beta.$$

It turns out that increasing returns to scale are equivalent $(1-\alpha)c'(\alpha)$ being decreasing in α. Constant returns to scale involve $(1-\alpha)c'(\alpha)$ being constant, which implies $c(\alpha)=-\theta\log(\alpha)$.

Theorem: There is a pure strategy equilibrium, which involves $\alpha_1>\alpha_2=\alpha_3=\ldots=\alpha_n$. If there are increasing return to scale, $\alpha_1>2\alpha_2$.

The closed form solution to the price dispersed equilibrium makes it a natural vehicle for industrial organization theory.