Price Discrimination

Shoe: Buy one, get one free

Price discrimination is charging different people different prices for the same good.

Student or senior citizen discounts

Coupons

Frequent flyer programs

Quantity discounts

- electricity
- phone service
- frequent flyer programs
- multi-packs of paper towels, lightbulbs, toothpaste, etc.
- shopping clubs
- outlet malls

Bargaining (personalized prices)
- automobiles
- third world

Damaged Goods
- student software
- Intel 486SX
- IBM LaserPrinter E
- Sony Minidisc
- Fedex 2ND day delivery

Freight absorption

It is not price discrimination to pass on cost savings.

VARIAN

Each consumer demands a single unit
Consumers are ranked on a continuum by their type t.
Distribution of types be F, and index types by their probability q=F(t).
The willingness to pay of a type t consumer is p(q), $p' < 0$.

A non-discriminating monopolist earns qp(q); let q_0 maximize profits.

A two price discriminating monopolist earns $q_1 p(q_1) + (q_2 - q_1) p(q_2)$ and let q_1 and q_2 stand for the maximing arguments.

Theorem (Varian 1985): Quantity and welfare (sum of profits and consumer surplus) are higher under price discrimination.

Proof: Note that welfare depends only on quantity.
Thus, it is sufficient to prove that quantity is not lower under price discrimination.
Suppose not, that is, suppose $q_2 < q_0$. Then

$$-p(q_0)q_1 > -p(q_2)q_1.$$

Profit maximization for the non-discriminating monopolist insures

$$p(q_0)q_0 \geq p(q_2)q_2.$$

Add these two inequalities to obtain

$$p(q_0)(q_0 - q_1) > p(q_2)(q_2 - q_1)$$

which implies

$$p(q_1)q_1 + p(q_0)(q_0 - q_1) > p(q_1)q_1 + p(q_2)(q_2 - q_1),$$

which contradicts profit maximization of the two price monopolist. Q.E.D.

Suppose there are n markets, and demand is given by $x_i(\mathbf{p})$ in market i where $\mathbf{p}=(p_1,\ldots,p_n)$.

$$\pi = \sum_{i=1}^{n} (p_i - mc)x_i(\mathbf{p}).$$

A non-discriminating monopolist charges a constant price p_0 in all n markets.

The discriminating monopolist will charge distinct prices p_i in the markets, $i=1,\ldots,n$.

Define the cross-price elasticity of substitution

$$\varepsilon_{ij} = \frac{p_j}{x_i}\frac{dx_i}{dp_j}.$$

Let E be the matrix of elasticities. Note that, if preferences can be expressed as the maximization of a representative consumer, then the consumer maximizes $u(\mathbf{x})-\mathbf{px}$, which gives FOC $u'(\mathbf{x}) = \mathbf{p}$, and thus $u''(x)\mathbf{dx} = \mathbf{dp}$. This shows that demand $\mathbf{x}$ has a symmetric derivative, a fact used in the next development.

The first order condition for profit maximization entails

$$0 = \frac{\partial \pi}{\partial p_i} = x_i + \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_j}{\partial p_i} = x_i + \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_i}{\partial p_j}$$

$$= x_i \left(1 + \sum_{j=1}^{n} \frac{(p_j - mc)}{p_j} \varepsilon_{ij} \right)$$

Let $L_i = \frac{p_i - mc}{p_i}$, and express the first order condition in a matrix format:

$\mathbf{0} = \mathbf{1} + E\,\mathbf{L}$, and thus $\mathbf{L} = -E^{-1}\,\mathbf{1}$. This generalizes the well-known one-good case of

$$\frac{p - mc}{p} = -\frac{1}{\varepsilon},$$

where å is the elasticity of demand (with a minus sign).

In the most frequently encountered version of monopoly pricing, demands are independent, in which case E is a diagonal matrix. The markets are then independent, and

$$\frac{p_i - mc_i}{p_i} = -\frac{1}{\varepsilon_{ii}}.$$

Theorem (Varian, 1985): The change in welfare, ΔW, when a monopolist goes from non-discrimination to discrimination is given by

$$\sum_{i=1}^{n} (p_i - mc)\Delta x_i \leq \Delta W \leq (p_0 - mc)\sum_{i=1}^{n} \Delta x_i.$$

Proof: Let $\mathbf{p_0} = p_0\mathbf{1}$, be the one-price monopoly price vector, and $\mathbf{p}$ represent the prices of the discriminating monopolist.

Let v be the indirect utility function (consumer utility as a function of prices). The indirect utility function is convex, and its derivative is demand (Roy's identity). Therefore,

$$\mathbf{x}(\mathbf{p_0})(\mathbf{p_0} - \mathbf{p}) \leq v(\mathbf{p}) - v(\mathbf{p_0}) \leq \mathbf{x}(\mathbf{p})(\mathbf{p_0} - \mathbf{p})$$

The change in profits is

$$\Delta\pi = \mathbf{x}(\mathbf{p})(\mathbf{p} - mc\,\mathbf{1}) - \mathbf{x}(\mathbf{p_0})\mathbf{1}(p_0 - mc)$$

Since the change in welfare is the change in consumer utility plus the change in profits, we have

$$\mathbf{x}(\mathbf{p_0})(\mathbf{p_0} - \mathbf{p}) + \Delta\pi \leq \Delta W \leq \mathbf{x}(\mathbf{p})(\mathbf{p_0} - \mathbf{p}) + \Delta\pi,$$

which combines with $\Delta\mathbf{x} = \mathbf{x}(\mathbf{p}) - \mathbf{x}(\mathbf{p_0})$ to establish the theorem. Q.E.D.

This theorem has a powerful corollary, first established by Schmalensee.

If price discrimination causes output to fall, then price discrimination decreases welfare relative to the absence of price discrimination.

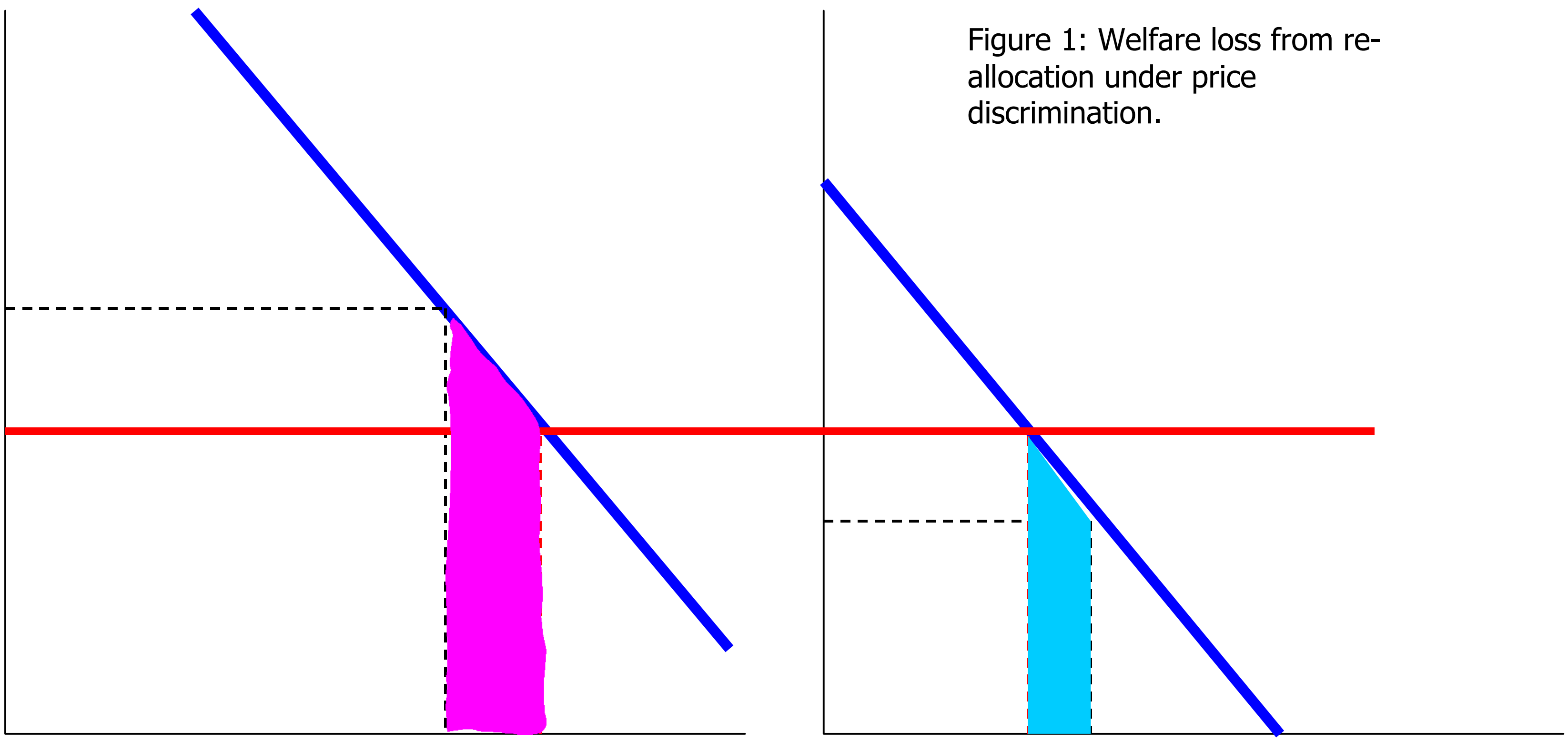

Figure 1: Welfare loss from re-allocation under price discrimination.

Market 1: pink area lost by price discrimination

Market 2: blue area added by discrimination

Even in the simplest two-market case of linear demand, price discrimination may increase or decrease welfare.

It is straightforward to construct cases where welfare rises under price discrimination.

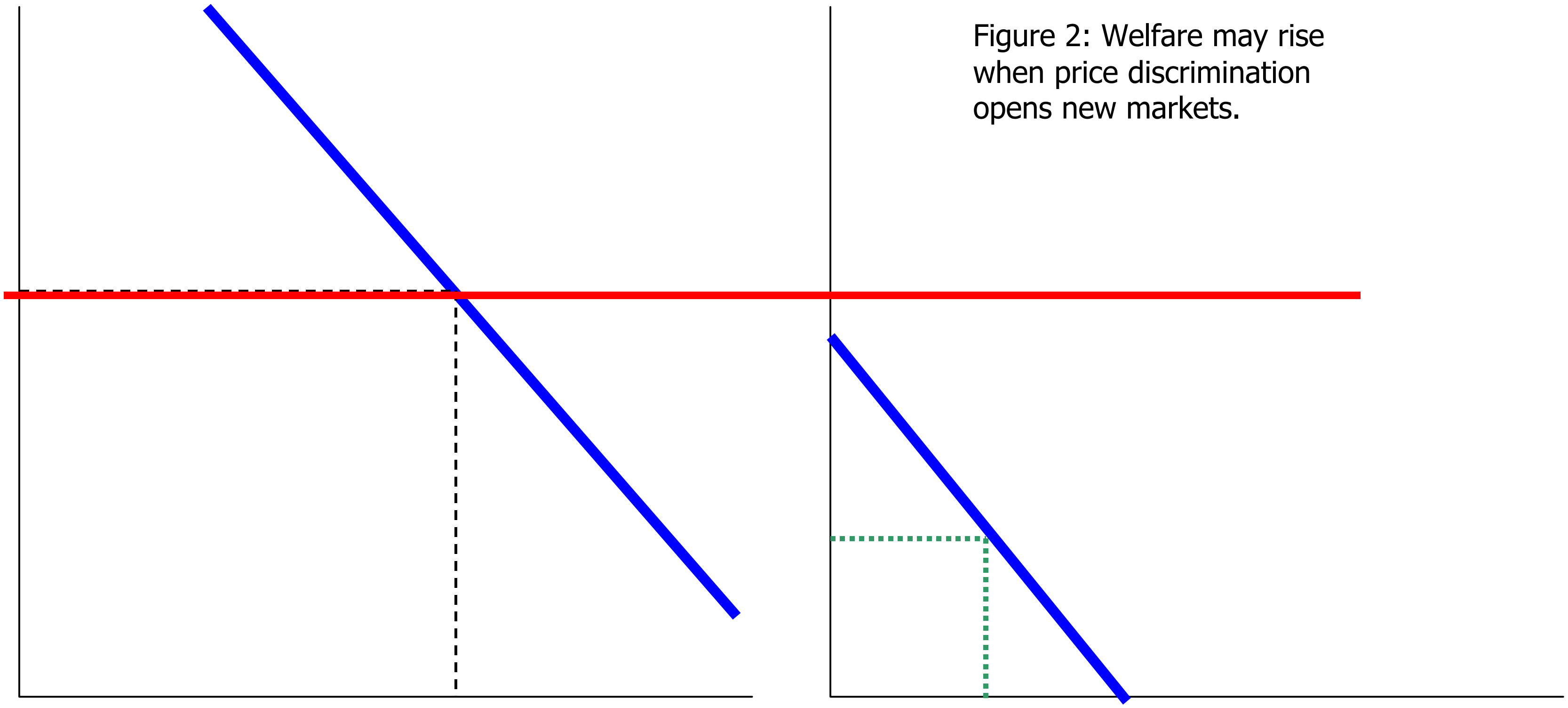

Figure 2: Welfare may rise when price discrimination opens new markets.

Market 1: Red line indicates no price discrimination outcome.

Market 2: With price discrimination, market 2 is served.

Ramsey Pricing

How should a multi-product or multi-market monopolist be regulated? Consider the problem

$$\max\ u(\mathbf{x}) - c(\mathbf{x1}) \quad \text{s.t. } \mathbf{px} - c(\mathbf{x1}) \geq \pi_0.$$

This formulation permits average costs to be decreasing. Write the Lagrangian

$$\Lambda = u(\mathbf{x}) - c(\mathbf{x1}) + \lambda(\mathbf{px} - c(\mathbf{x1})) = u(\mathbf{x}) - \mathbf{px} + (1+\lambda)(\mathbf{px} - c(\mathbf{x1}))$$

The lagrangian term λ has the interpretation that it is the marginal increase in welfare associated with a decrease in firm profit. Using Roy's identity,

$$0 = \frac{\partial L}{\partial p_i} = \lambda x_i + (1+\lambda)\sum_{j=1}^{n}(p_j - mc)\frac{\partial x_j}{\partial p_i} = \lambda x_i + (1+\lambda)\sum_{j=1}^{n}(p_j - mc)\frac{\partial x_i}{\partial p_j}$$

$$= \lambda x_i + (1+\lambda)x_i \sum_{j=1}^{n}\frac{p_j - mc}{p_j}\varepsilon_{ij}.$$

Write the first order conditions in vector form, to obtain

$-\lambda/(1+\lambda)\ \mathbf{1} = E\ \mathbf{L}$.

This equation solves for the general Ramsey price solution:

$$\mathbf{L} = -\frac{\lambda}{\lambda+1}\mathbf{E}^{-1}\mathbf{1}.$$

The monopoly outcome arises when $\lambda \to \infty$.

Setting $\lambda = 0$ maximizes total welfare and sets price equal to marginal cost in all industries.

Arbitrage

Cross-price elasticities can be interpreted as a consequence of arbitrage by individuals.
Suppose leakage from the low priced market to the high priced market costs c(m), where m is the size of the transfer from market 1 to market 2, and that values in the two markets are otherwise independent.

The function c is assumed convex, with $c'(0) = 0$, which insures that goods flow from the low priced market to the high priced market.
Assume that consumer demands in markets 1 and 2 are $q_1(p_1)$ and $q_2(p_2)$. The demands facing the seller, x_i will satisfy:

$$p_1 - p_2 = c'(m),$$
$$q_1(p_1) - m = x_1, \text{ and}$$
$$q_2(p_2) + m = x_2,$$

An interesting aspect of these equations is that demand is reconcilable with preferences of a single consumer, that is: $\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i}$.

Means of Preventing Arbitrage

1. Services
2. Warranties
3. Differentiating products
4. Transport costs
5. Contracts
6. Matching problem
7. Government
8. Quality