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Investment decisions under first and second price auctions

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A seller of an indivisible object faces multiple buyers, whose valuations are a function of their unobservable prior investments. A second price auction generates efficient investment, but the seller prefers a first price sealed bid auction, which induces inefficient investment.

1. Introduction

In this paper we consider settings in which there is one seller of an indivisible object and many potential buyers. The seller can choose to conduct either a first or second price sealed bid auction to allocate the object. The value of the object to a buyer is an increasing function of some unobservable investment, the level of which is chosen by the buyer before submitting a bid. We demonstrate that the choices of the buyers depend upon the type of auction chosen by the seller. If the seller runs a sealed bid *second* price auction, the equilibrium involves asymmetric investments: only one buyer invests, and this buyer chooses the efficient level of investment and wins the object. If the seller chooses a sealed bid *first* price auction, a symmetric, mixed strategy, inefficient equilibrium emerges. The seller prefers the latter outcome, while the buyers prefer the former. Thus, if the seller can choose the type of auction to hold, in the resulting equilibrium net expected surplus will not be maximized.

This result is relevant for several economic environments where valuations of objects are endogenous. In the case of government procurement of goods and services, the government is a seller of procurement contracts, which firms buy. Firms can direct research and development efforts to lower production costs, and hence increase the return obtainable from a given contract. [1]

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[1] For a discussion of some existing procedures in government purchasing of both goods and services, see McAfee and McMillan (1988) and Avio (1992).

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Local governments considering bidding for large-scale industrial investment in their jurisdictions can invest in local infrastructure. [2] In both cases the seller's preferred auction induces levels of investment which are inefficient, although the object is sold to the more efficient buyer.

2. The model

Let x_i denote the level of investment chosen by buyer i, $i = \text{A}, \text{B}$. For convenience, we assume that one unit of investment yields one unit of gross surplus, so x_i will also denote the value produced by buyer i when the buyer has invested this amount. We assume the buyers are initially identical. Investment is produced according to an increasing, strictly convex cost function $\gamma(x_i)$; this cost function is common to both buyers. Let b_i denote the bid submitted by buyer i, $i = \text{A}, \text{B}$; $\pi_i(x_i) = \pi(b_\text{A}, b_\text{B}, x_i)$, the surplus retained by buyer i; and x^*, the efficient level of investment defined by $\gamma'(x^*) = 1$. Since only one of the buyers will be chosen, without loss of generality we let A be the winning buyer.

2.1. Second price or oral auction

If the seller holds an oral auction, the actual levels of the x's will be revealed during the auction, [3] and the expected surplus accruing to buyer A is

$$\pi_\text{A}(x_\text{A}) = \begin{cases} x_\text{A} - x_\text{B} - (x_\text{A}) & \text{if} \quad x_\text{A} \geq x_\text{B}, \\ -\gamma(x_\text{A}) & \text{if} \quad x_\text{A} < x_\text{B}. \end{cases} \tag{1}$$

The buyer's choice of investment is then *either* $x_\text{A}^* = x^*$, *or* $x_\text{A}^* = 0$, since

$$\partial\pi_\text{A}(x_\text{A})/\partial x_\text{A} = \begin{cases} 1 - \gamma'(x_\text{A}) & \text{if} \quad x_\text{A} \geq x_\text{B}, \\ -\gamma'(x_\text{A}) & \text{if} \quad x_\text{A} < x_\text{B}. \end{cases} \tag{2}$$

Buyer B's optimal strategy can be derived in a similar fashion. These results are summarized in the following lemma.

Lemma 1. The oral auction generates an asymmetric pure strategy equilibrium in which one buyer chooses the efficient level of investment, and wins the auction, while the other buyer invests nothing.

A sealed bid second price auction would yield the same result in this game.

[2] In King and Welling (1990) we consider two local governments competing to attract a plant. In King et al. (1991) we extend the analysis to include prior investment by local governments. Both papers assume symmetric information. First and second price auctions yield the same allocations in these settings.

[3] See Milgrom and Weber (1982) for features of the oral and sealed bid, second price auctions in this context. McAfee and McMillan (1987) provide a general survey of auction theory.

2.2. First price sealed bid auction

In the first price auction, consider buyer A's pay-off, if it bids b and buyer B bids b_B. Then buyer A receives

$$\pi_A(x_A) = \begin{cases} x_A - b - \gamma(x_A) & \text{if} \quad b > b_B, \\ -\gamma(x_A) & \text{if} \quad b < b_B, \\ 0.5(x_A - b) - \gamma(x_A) & \text{if} \quad b = b_B. \end{cases} \tag{3}$$

A preliminary result is given in Lemma 2:

Lemma 2. There is no pure strategy equilibrium in the first price sealed bid auction.

Proof. (a) Consider a symmetric pure strategy equilibrium where $b = b_B$. If $x_A > b$, then $b = b_B + \epsilon$ produces a higher pay-off for $\epsilon < (x_A - b)/2$. If $x_A = b = b_B$, then $\pi_A(x_A) < 0$. Since $\pi_A(0) = 0$, this is not an equilibrium. If $x_A = 0$, then $x_B = 0$ by symmetry. But then the choice $x_A = x^*$, and $b = \epsilon$ yields a higher pay-off. Therefore $b = b_B$ is not an equilibrium, so no symmetric pure strategy equilibrium exists.
(b) In any asymmetric pure strategy equilibrium, one buyer loses. Let that be buyer B. Given that it loses, $x_B = 0$. Suppose it bids b_B. Since buyer A wins, it invests

$$x_A = x^* = \operatorname{argmax}[x - b - \gamma(x)]$$

and earns $\pi_A(x^*) = x^* - b - \gamma(x^*) < x^* - b_B - \gamma(x^*)$.

(i) If $b_B < x^* - \gamma(x^*)$, buyer A wants to bid $b_A = \min\{b > b_B\}$, which does not exist.
(ii) If $b_B \geq x^* - \gamma(x^*)$, buyer A earns $\pi_A(x^*) < x^* - (x^* - \gamma(x^*)) - \gamma(x^*) = 0$, which cannot be an equilibrium.

Therefore no pure strategy equilibrium exists. □

Although there is no pure strategy equilibrium to the investment game in the first price sealed bid auction, the next lemma proves the existence of a symmetric mixed strategy equilibrium. Since the socially efficient outcome has one buyer choosing the optimal level of investment, and the other choosing zero, and equilibrium in which buyers randomize their choices of investment is not efficient.

Lemma 3. There exists a symmetric mixed strategy equilibrium when the seller holds a sealed bid, first price auction. Each buyer randomizes over [0, x^*].

Proof. Suppose buyer B bids $\leq b$ with probability $F(b)$. Then buyer A expects to receive $\pi_A(x_A, b) = (x_A - b)F(b) - \gamma(x_A)$. In a mixed strategy, one bid, say $\underline{b}$, loses with probability one. Thus $\pi_A(x_A, \underline{b}) = -\gamma(x_A) \leq 0$. Therefore $x_A = 0$, and $\pi_A(0, b) = 0$. This implies $\pi_A(x, b) = 0$ over all bids that occur. Thus buyer A earns $(x - b)F(b) - \gamma(x) = 0$; the optimal x given a chosen bid b satisfies $0 = F(b) - \gamma'(x)$.

If $B(x)$ is the bidding function, then

$$0 = [x - B(x)]F(B(x)) - \gamma(x) = [x - B(x)]\gamma'(x) - \gamma(x).$$

Thus

$$B(x) = x - \frac{\gamma(x)}{\gamma'(x)}, \quad \text{and} \quad B'(x) = \frac{\gamma(x)\gamma''(x)}{\gamma'(x)^2} > 0,$$

since $\gamma(\cdot)$ is strictly convex.

Thus $F(\cdot)$ is implicitly defined by

$$\gamma'(x) = F(B(x)) = F\left(x - \frac{\gamma(x)}{\gamma'(x)}\right).$$

Since $\gamma'(x^*) = 1$, $F(B(x^*)) = F(x^* - \gamma(x^*)) = 1$, and x ranges from 0 to x^*. The cdf of x is $\gamma'(x)$; to see this, note that

$$\text{Prob}(x \leq \hat{x}) = \text{Prob}(B(x) \leq B(\hat{x})) = F(B(x)) = \gamma'(x). \qquad \square$$

Lemmas 1 and 3 demonstrate that the second price auction yields results which are superior if the goal is to maximize expected surplus. However, if the seller can choose which auction to run, it may choose the one which maximizes its expected return. The conflict between the social optimum and the seller's goal is demonstrated in Lemma 4.

Lemma 4. The seller's surplus under the first price sealed bid auction exceeds that obtainable with a second price auction.

Proof. In the second price auction, one buyer chooses zero investment. Since the seller's return cannot exceed the maximum of its reservation price or the bid from the losing buyer, all of the surplus generated by investment accrues to the winning buyer. In the first price auction, each buyer's expected equilibrium pay-off is zero. Since the equilibrium is symmetric,

$$\text{expected surplus} = 2\int_0^{x^*} [xF(B(x)) - \gamma(x)]\gamma''(x)\,\mathrm{d}x.$$

The term in brackets must be strictly positive for any x for which a buyer submits a positive bid. All of this surplus is captured by the seller. $\square$

3. Conclusion

We have demonstrated that when bidders' valuations are endogenous and unobservable, the type of auction chosen has efficiency effects: there is a conflict between maximizing the social surplus and maximizing the expected pay-off to the seller of the good being auctioned. In the case of government contracting, these results imply that a government agency which commits to an auction form which maximizes its expected return (rather than the social surplus) will choose a buyer efficiently given the pre-determined investments, but will induce an inefficient level of investment on the part of firms interested in winning the contract. In the case of local governments engaged in bidding wars for plants, similar results apply: although the firm's location decision will be efficient, given the levels of infrastructure chosen by the bidding regions, the regional governments will not choose the efficient levels of investment when the firm can choose (and commit to) the auction form.

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