Peak Load Pricing

How should capacity be priced?

- Pipelines
- Airlines
- Telephone networks
- Construction
- Electricity
- Highways
- Internet

Pioneered by Marcel Boiteaux

$$\pi = p_1 q_1 + p_2 q_2 - \beta \max\{q_1, q_2\} - mc(q_1 + q_2).$$

Social welfare is

$$W = \int_0^{q_1} p_1(x)dx + \int_0^{q_2} p_2(x)dx - \beta \max\{q_1, q_2\} - mc(q_1 + q_2).$$

The Ramsey problem is to maximize W subject to a profit condition. As always, write the lagrangian $L = W + \lambda\,\pi$.

$$0 = \frac{\partial L}{\partial q_1} = p_1(q_q) - \beta 1_{q_1 \geq q_2} - mc + \lambda\left(p_1(q_q) + q_1 p_1'(q_1) - \beta 1_{q_1 \geq q_2} - mc\right)$$

Or,

$$\frac{p_1(q_1) - \beta 1_{q_1 \geq q_2} - mc}{p_1} = \frac{\lambda}{\lambda + 1}\frac{1}{\varepsilon_1}$$

where $1_{q_1 \geq q_2}$ is the characteristic function of the event $q_1 \geq q_2$.

Similarly,

$$\frac{p_2(q_2) - \beta 1_{q_1 \le q_2} - mc}{p_2} = \frac{\lambda}{\lambda + 1} \frac{1}{\varepsilon_2}$$

Note as before that $\lambda \to \infty$ yields the monopoly solution.

There are two potential types of solution.

Let the demand for good 1 exceed the demand for good 2.

Then either $q_1 > q_2$, or the two are equal.

Case 1: $q_1 > q_2$.

$$\frac{p_1(q_1) - \beta - mc}{p_1} = \frac{\lambda}{\lambda + 1}\frac{1}{\varepsilon_1} \text{ and } \frac{p_2(q_2) - mc}{p_2} = \frac{\lambda}{\lambda + 1}\frac{1}{\varepsilon_2}.$$

In case 1, with all of the capacity charge allocated to good 1, quantity for good 1 still exceeds quantity for good 2.

Thus, the peak period for good 1 is an extreme peak.

Case 2: $q_1 = q_2$.

The first order conditions become inequalities, of the form

$$0 \le p_1(q_q) - mc + \lambda\left(p_1(q_q) + q_1 p_1'(q_1) - mc\right) \le (1+\lambda)\beta.$$

$$0 \le \frac{p_1(q_1) - mc}{p_1} - \frac{\lambda}{\lambda+1}\frac{1}{\varepsilon_1} \le \beta \text{ and } 0 \le \frac{p_2(q_2) - mc}{p_2} - \frac{\lambda}{\lambda+1}\frac{1}{\varepsilon_2} \le \beta.$$

These must solve at $q_1 = q_0 = q$. The profit equation can be written

$p_1(q)$ - $mc + p_2(q) - mc = \beta$

This equation shows that the capacity charge is shared across the two markets proportional to the inverse demand.

Not shared according to elasticities!

Priority Pricing

Consider a case of a continuum of consumers, each of whom desires one unit.

Rank the consumers by their valuations for the good, so that the q^{th} consumer has a value $p(q)$ for the good, and p is downward sloping.

The quantity available is a random variable with distribution F.

Priority pricing is a charge schedule c which provides a unit to a customer paying $c(q)$ whenever realized supply is q or greater.

A customer of type q should choose to pay $c(q)$ for the q^{th} spot in the priority list. This leads to the incentive constraint:

$$u(q) = (p(q) - c(q))(1 - F(q)) \geq (p(q) - c(\hat{q}))(1 - F(\hat{q})).$$

The envelope theorem gives

$$u'(q) = p'(q)(1 - F(q)).$$

It is a straightforward exercise to demonstrate that the first order condition is sufficient; see handout #2.

Let $F(H)=1$, so that $u(H)=0$. Then

$$(p(q)-c(q))(1-F(q)) = u(q) = -\int_q^H u'(s)ds = -\int_q^H p'(s)(1-F(s))ds$$

$$= p(q)(1-F(q)) - \int_q^H p(s)f(s)ds$$

Thus,

$$c(q) = \int_q^H p(s)\frac{f(s)}{1-F(q)}ds = E[spot\ \ price \mid p(s) \geq p(q)].$$

Revenues to the firm from the priority pricing are

$$R = \int_0^H c(q)(1-F(q))dq = \int_0^H \int_q^H p(s)f(s)\,ds\,dq = \int_0^H qp(q)f(q)\,dq.$$

This is the revenue associated with a competitive supply;

A monopolist might have an incentive to withhold capacity to boost prices.

How does a monopolist do so? Withholding of capacity has the property of changing the distribution of available supply, in a first order stochastic dominant manner. In particular, the monopolist can offer any distribution of capacity *G*, provided $G \geq F$. What is the monopolist's solution? Rewrite *R* to obtain

$$R = \int_0^H qp(q)g(q)dq = \int_0^H MR(q)(1 - G(q))dq.$$

Provided marginal revenue *MR* is single-peaked,

$$G = \begin{cases} F \text{ } if \text{ } MR \geq 0 \\ \\ 1 \text{ } if \text{ } MR < 0 \end{cases}.$$

That is, the monopolist cuts off the capacity at the monopoly supply.

Matching Problems

Consider first the linear demand case with a uniform distribution of outages. Perfect matching gets a payoff

$$\int_0^1 p(q)(1-q)dq = \int_0^1 (1-q)^2 dq = \frac{1}{3}.$$

No matching – that is a random assignment – produces an expected value of ¼, a fact that is evident from

$$\int_0^1 p(q)dq \int_0^1 (1-q)dq = \left(\int_0^1 (1-q)dq \right)^2 = \frac{1}{4}.$$

Now consider two groups of equal size.

The high value group has an average value of ¾, and is served with probability $\int_0^{1/2} 2q\,dq + \int_{1/2}^{1} 1\,dq = ¾$.

The low value group has average value ¼ and is served with probability 1/4. Thus, the expected value from two categories is

$$\frac{1}{2}\left(\frac{9}{16}+\frac{1}{16}\right)=\frac{5}{16}.$$

Note that 5/16 is 75% of the way from ¼ to 1/3! That is, a single group captures 75% of net value of a continuum of types!

I show elsewhere that, provided a common hazard rate assumption is satisfied, two groups of equal size generally captures 50% or more of the possible gains over no priority pricing.

Wilson shows that the losses from finite classes are on the order of $1/n^2$.