The Two Type Model

Consumer L, for low type, with value $v_L(q)$ for quantity q, and H, for high type, with value v_H. Both value nothing at zero, so $v_L(0)=v_H(0)=0$.

(1) $v'_H(q) \geq v'_L(q)$.

The monopolist offers two quantities q_L and q_H at prices R_L and R_H, respectively, targeted to the consumers L and H.

(IR_L) $\quad v_L(q_L)-R_L \geq 0$

(IR_H) $\quad v_H(q_H)-R_H \geq 0$.

(IC_L) $\quad v_L(q_L)-R_L \geq v_L(q_H)-R_H$

(IC_H) $\quad v_H(q_H)-R_H \geq v_H(q_L)-R_L$.

The monopolist is assumed to have a constant marginal cost c, and to maximize profit $R_L + R_H - c(q_L + q_H)$.

Claim 1: $q_L \leq q_H$.

Proof: Rearrange IC_L and IC_H to obtain

$$v_H(q_H) - v_H(q_L) \geq R_H - R_L \geq v_L(q_H) - v_L(q_L).$$

This gives

$$\int_{q_L}^{q_H} v'_H(q)\ dq = v_H(q_H) - v_H(q_L) \geq v_L(q_H) - v_L(q_L)$$

$$= \int_{q_L}^{q_H} v'_L(q)\ dq,$$

or, $$\int_{q_L}^{q_H} v'_H(q) - v'_L(q)\ dq \geq 0,$$

from which (1) proves the claim.

Claim 2: IR_H can be ignored. That is, IC_H and IR_L imply IR_H.

Proof: Using first IC_H then IR_L, note that

$$v_H(q_H) - R_H \geq v_H(q_L) - R_L = \int_0^{q_L} v_H'(q)\ dq - R_L$$

$$\geq \int_0^{q_L} v_L'(q)\ dq - R_L = v_L(q_L) - R_L \geq 0.$$

Thus, if IC_H and IR_L are satisfied, then IR_H is automatically satisfied, and can be ignored.

Claim 3: IC_H is satisfied with equality at the monopolist's profit maximization.

Claim 4: IR_L holds with equality.

Claims 3 and 4 let us express the monopolists objective function in terms of the quantities, merely by using the constraints that hold with equality. That is,

$R_L + R_H - c(q_L+q_H) = 2v_L(q_L) + v_H(q_H) - v_L(q_H) - c(q_L+q_H)$.

This gives the first order conditions

$$0 = v_H'(q_H) - c,$$

and

$$0 = 2v_L'(q_L) - v_H'(q_L) - c.$$

The second may not be satisfiable, and in fact, if the demand of the high type is twice or more the demand of the low type, that is, $v_H'(q) > 2v_L'(q)$, then the monopolist's optimal quantity $q_L=0$, and the low type is shut out of the market.

Implications

1. The high type gets the "efficient" quantity (i.e. the quantity that a benevolent social planner would award him.

2. The low type gets strictly less than the efficient quantity.

3. The high type has a positive consumer surplus, that is, $v_H(q_H)-R_H>0$, unless $q_L=0$.

4. The low type gets zero consumer surplus.

The Continuum Model

Consumers have utility $v(q,t)$ - p, where t is the type in $[0,1]$ with density $f(t)$, q is quantity and p is the payment made.

The monopolist will set an aggregate charge $R(q)$ for the purchase of q. What should the schedule of prices $R(q)$ look like?

Define the shadow price $p(q,t) = v_q(q,t)$, which gives the demand curve of the type t.

Assume $p_t(q,t) > 0$, that is, higher types have higher demands, and that $v(0,t) = 0$.

We will look for a function q*(t) so that a type t agent purchases q*(t). Any candidate function q(t) must satisfy

(IC) $v(q(s),t)-R(q(s)) \leq v(q(t),t) - R(q(t)) = \pi(t)$

yielding the first order condition

$$v_q(q(t),t) - R'(q(t)) = 0.$$

and (envelope theorem)

$$\pi'(t) = v_t(q(t),t).$$

As before, the individual rationality constraint requires

(IR) $\pi(t) \geq 0$.

π is nondecreasing: $v_t(q,t) = \int_0^q v_{qt}(x,t)dx = \int_0^q p_t(x,t)dx \geq 0$

IR is equivalent to $\pi(0) \geq 0$.

Therefore,

$$\int_0^1 \pi(t)f(t)dt = -\pi(t)(1-F(t))\Big|_0^1 + \int_0^1 \pi'(t)(1-F(t))dt$$

$$-\pi(0) + \int_0^1 v(q(t),t)(1-F(t))dt$$

Consequently, the monopolist's profit can be expressed as:

$$\int_0^1 R(q(t)) - cq(t))f(t)dt = \int_0^1 (v(q(t),t) - \pi(t) - cq(t))f(t)dt$$

$$= -\pi(0) + \int_0^1 \left(v(q(t),t) - \frac{1-F(t)}{f(t)} v_t(q(t),t) - cq(t) \right) f(t)dt.$$

Maximizing pointwise gives:

(2) $\quad p(q^*(t),t) - \dfrac{1-F(t)}{f(t)} p_t(q^*(t),t) - c = 0.$

Necessity and sufficiency: The IC constraint holds if and only if the first order condition for the buyer's maximization holds, and q is nondecreasing

Let $u(s,t)=v(q(s),t) - R(q(s))$, which is what a type t agent gets if he buys the quantity slated for type s. Then IC can be written

$$u(s,t) \leq u(t,t).$$

Denote partial derivatives with subscripts. Necessarily, $u_1(t,t)=0$ and $u_{11}(t,t)\leq 0$. Totally differentiating the first gives $u_{11}(t,t) + u_{12}(t,t)=0$, so the second order condition can be rewritten $u_{12}(t,t)\geq 0$. Therefore, necessarily,

$$0 \leq v_{qt}(q(t),t)q'(t),$$

which forces q nondecreasing, since $v_{qt}=p_t>0$. Now turn to sufficiency. Note that, if q is nondecreasing, then u_{12} is everywhere nonnegative. Thus, for $s<t$, $u_1(s,t) \leq u_1(s,s) = 0$, and for $s>t$, $u_1(s,t) \geq u_1(s,s) = 0$. Thus, u is increasing in s for $s<t$, and decreasing in s for $s>t$, and therefore u is maximized at $s=t$, and IC holds.

Thus, $q^{*\prime}(t) \geq 0$ is both necessary and sufficient for the solution to

$$R'(q^*(t)) = p(q^*(t),t)$$

$$R(q^*(0)) = v(q^*(0),0)$$

to maximize the monopolist's profit, where q^* is given by (2). This defines the optimal R.

Observations:

(1) The highest type consumer gets the efficient quantity, in that price p(q*(1),1) = c, marginal cost

(2) Those with greater demand (high t's, since $p_t>0$) obtain at least as much of the good, and sometimes more, than those with lower demand.

(3) All agents except the highest type get less than the efficient quantity

This follows from

$$p(q^*(t),t) - c = \frac{1-F(t)}{f(t)} p_t(q^*(t),t) > 0,$$

since

$$v_t(q,t) = \int_0^q v_{qt}(r,t)dr > 0.$$

(4) If the optimal quantity is decreasing in some neighborhood, then a flat spot results from the optimization and an interval of types are treated equally (called pooling).

(5) The monopolist's solution may be implemented using a nonlinear price schedule. Under some circumstances, it may be implemented using a menu of linear price schedules, that is, offering lower marginal costs, at a higher fixed cost, much like phone companies do.

(6) The solution can be interpreted according to the elasticity formula already given. Let $y = 1 - F(t)$ represent the number of consumers willing to buy q(t) at price p(q(t),t). Note that

$$\frac{p(q(t),t) - c}{p(q(t),t)} = \frac{1 - F(t)}{f(t)} \frac{p_t(q(t),t)}{p(q(t),t)} = \frac{\frac{\partial}{\partial t}\log(p(q,t))}{-\frac{\partial}{\partial t}\log(1 - F(t))} = -\frac{1}{\frac{p}{y}\frac{dy}{dp}}.$$

Quality Premia

Two types of consumers, L and H.

Both types value higher quality more, but the H type values an increase in quality more than the L type, that is, $v'_L(q) < v'_H(q)$.

In this case, the monopolist will offer two qualities, one high and one low.

The high quality good will be efficient, i.e. sets the marginal value of quality to the marginal cost.

The low quality, however, will be worse than efficient. That is, the monopolist will intentionally make the low quality good worse, so as to be able to charge more for the high quality good.

The case where c=0 is especially interesting, because this is the case in which quality is free, say, up to an upper bound $\bar{q}$.

One can imagine that the monopolist only produces one good, and, at no cost, can make it lower quality, say, by hitting it with a hammer.

In this case, the monopolist will still offer two qualities, that is, the monopolist will intentionally damage a portion of the goods he sells, so as to be able to segment the market.

Tie-ins

Tie-ins arise whenever a manufacturer requires the purchase of one product in order to purchase another product.

Reasons for Tie-ins

1. Lower Cost

   a. save on packaging
   b. save on sorting

2. Evade price controls

3. Circumvent other regulation

4. Offer Secret Price Cuts

5. Assure Quality

6. Price Discriminate