Price Discrimination

Shoe: Buy one, get one free

Price discrimination is charging different people different prices for the same good.

Student or senior citizen discounts

Coupons

Frequent flyer programs

Quantity discounts
- electricity
- phone service
- frequent flyer programs
- multi-packs of paper towels, lightbulbs, toothpaste, etc.
- shopping clubs
- outlet malls
Bargaining (personalized prices)
- automobiles
- third world

Damaged Goods
- student software
- Intel 486SX
- IBM LaserPrinter E
- Sony Minidisc
- Fedex 2\textsuperscript{ND} day delivery

Freight absorption

It is not price discrimination to pass on cost savings.
VARIAN

Each consumer demands a single unit
Consumers are ranked on a continuum by their type $t$.
Distribution of types be $F$, and index types by their probability $q = F(t)$.
The willingness to pay of a type $t$ consumer is $p(q)$, $p' < 0$.

A non-discriminating monopolist earns $qp(q)$; let $q_0$ maximize profits.

A two price discriminating monopolist earns $q_1p(q_1) + (q_2-q_1)p(q_2)$ and let $q_1$ and $q_2$ stand for the maximising arguments.

Theorem (Varian 1985): Quantity and welfare (sum of profits and consumer surplus) are higher under price discrimination.
Proof: Note that welfare depends only on quantity. Thus, it is sufficient to prove that quantity is not lower under price discrimination. Suppose not, that is, suppose $q_2 < q_0$. Then

$$-p(q_0)q_1 > -p(q_2)q_1.$$ 

Profit maximization for the non-discriminating monopolist insures

$$p(q_0)q_0 \geq p(q_2)q_2.$$ 

Add these two inequalities to obtain

$$p(q_0)(q_0 - q_1) > p(q_2)(q_2 - q_1)$$

which implies

$$p(q_1)q_1 + p(q_0)(q_0 - q_1) > p(q_1)q_1 + p(q_2)(q_2 - q_1),$$

which contradicts profit maximization of the two price monopolist. Q.E.D.
Suppose there are $n$ markets, and demand is given by $x_i(p)$ in market $i$ where $p = (p_1, \ldots, p_n)$.

$$\pi = \sum_{i=1}^{n} (p_i - mc) x_i(p).$$

A non-discriminating monopolist charges a constant price $p_0$ in all $n$ markets.

The discriminating monopolist will charge distinct prices $p_i$ in the markets, $i=1,\ldots,n$.

Define the cross-price elasticity of substitution

$$\varepsilon_{ij} = \frac{p_j}{x_i} \frac{dx_i}{dp_j}.$$

Let $E$ be the matrix of elasticities. Note that, if preferences can be expressed as the maximization of a representative consumer, then the consumer maximizes $u(x)-px$, which gives FOC $u'(x) = p$, and thus $u''(x)dx = dp$. This shows that demand $x$ has a symmetric derivative, a fact used in the next development.
The first order condition for profit maximization entails

\[ 0 = \frac{\partial \pi}{\partial p_i} = x_i + \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_j}{\partial p_i} = x_i + \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_i}{\partial p_j} \]

\[ = x_i \left( 1 + \sum_{j=1}^{n} \frac{(p_j - mc)}{p_j} \varepsilon_{ij} \right) \]

Let \( L_i = \frac{p_i - mc}{p_i} \), and express the first order condition in a matrix format:

\[ 0 = 1 + E \mathbf{L}, \] and thus \( \mathbf{L} = -E^{-1} \mathbf{1} \). This generalizes the well-known one-good case of

\[ \frac{p - mc}{p} = -\frac{1}{\varepsilon}, \]

where \( \varepsilon \) is the elasticity of demand (with a minus sign).
In the most frequently encountered version of monopoly pricing, demands are independent, in which case $E$ is a diagonal matrix. The markets are then independent, and

$$\frac{p_i - mc_i}{p_i} = -\frac{1}{\varepsilon_{ii}}.$$

Theorem (Varian, 1985): The change in welfare, $\Delta W$, when a monopolist goes from non-discrimination to discrimination is given by

$$\sum_{i=1}^{n} (p_i - mc) \Delta x_i \leq \Delta W \leq (p_0 - mc) \sum_{i=1}^{n} \Delta x_i.$$
Proof: Let $p_0 = p_0 \mathbf{1}$, be the one-price monopoly price vector, and $p$ represent the prices of the discriminating monopolist.

Let $v$ be the indirect utility function (consumer utility as a function of prices). The indirect utility function is convex, and its derivative is demand (Roy’s identity). Therefore,

$$x(p_0)(p_0 - p) \leq v(p) - v(p_0) \leq x(p)(p_0 - p)$$

The change in profits is

$$\Delta \pi = x(p)(p - mc \mathbf{1}) - x(p_0)\mathbf{1}(p_0 - mc)$$

Since the change in welfare is the change in consumer utility plus the change in profits, we have

$$x(p_0)(p_0 - p) + \Delta \pi \leq \Delta W \leq x(p)(p_0 - p) + \Delta \pi,$$

which combines with $\Delta x = x(p) - x(p_0)$ to establish the theorem. Q.E.D.
This theorem has a powerful corollary, first established by Schmalensee.

If price discrimination causes output to fall, then price discrimination decreases welfare relative to the absence of price discrimination.

Figure 1: Welfare loss from re-allocation under price discrimination.

Market 1: pink area lost by price discrimination

Market 2: blue area added by discrimination
Even in the simplest two-market case of linear demand, price discrimination may increase or decrease welfare.

It is straightforward to construct cases where welfare rises under price discrimination.

Market 1: Red line indicates no price discrimination outcome.

Market 2: With price discrimination, market 2 is served.

Figure 2: Welfare may rise when price discrimination opens new markets.
Ramsey Pricing

How should a multi-product or multi-market monopolist be regulated? Consider the problem

$$\max u(x) - c(x1) \quad \text{s.t.} \quad px - c(x1) \geq \pi_0.$$  

This formulation permits average costs to be decreasing. Write the Lagrangian

$$\Lambda = u(x) - c(x1) + \lambda(px - c(x1)) = u(x) - px + (1 + \lambda)(px - c(x1))$$

The lagrangian term $\lambda$ has the interpretation that it is the marginal increase in welfare associated with a decrease in firm profit. Using Roy’s identity,

$$0 = \frac{\partial L}{\partial p_i} = \lambda x_i + (1 + \lambda) \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_j}{\partial p_i} = \lambda x_i + (1 + \lambda) \sum_{j=1}^{n} (p_j - mc) \frac{\partial x_i}{\partial p_j}$$

$$= \lambda x_i + (1 + \lambda)x_i \sum_{j=1}^{n} \frac{p_j - mc}{p_j} \varepsilon_{ij}.$$
Write the first order conditions in vector form, to obtain

\[-\frac{\lambda}{(1+\lambda)} \mathbf{1} = E \mathbf{L}.\]

This equation solves for the general Ramsey price solution:

\[\mathbf{L} = -\frac{\lambda}{\lambda + 1} E^{-1} \mathbf{1}.\]

The monopoly outcome arises when \(\lambda \to \infty\).

Setting \(\lambda = 0\) maximizes total welfare and sets price equal to marginal cost in all industries.
Arbitrage

Cross-price elasticities can be interpreted as a consequence of arbitrage by individuals. Suppose leakage from the low priced market to the high priced market costs $c(m)$, where $m$ is the size of the transfer from market 1 to market 2, and that values in the two markets are otherwise independent.

The function $c$ is assumed convex, with $c'(0) = 0$, which insures that goods flow from the low priced market to the high priced market. Assume that consumer demands in markets 1 and 2 are $q_1(p_1)$ and $q_2(p_2)$. The demands facing the seller, $x_i$ will satisfy:

$$p_1 - p_2 = c'(m),$$
$$q_1(p_1) - m = x_1,$$
$$q_2(p_2) + m = x_2,$$

An interesting aspect of these equations is that demand is reconcilable with preferences of a single consumer, that is: $\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i}$. 
Means of Preventing Arbitrage

1. Services
2. Warranties
3. Differentiating products
4. Transport costs
5. Contracts
6. Matching problem
7. Government
8. Quality