Handout #5: The Coase Conjecture

In the paper "Durability and Monopoly," Nobel Laureate Ronald Coase proposes the startling hypothesis that the monopoly seller of a durable good will tend to price at marginal cost, absent some mechanism for committing to withhold supply. (Such mechanisms include leasing rather than selling, planned obsolescence, increasing marginal cost (which makes delay rational), and promises to repurchase at a fixed price.) The logic takes three steps. First, having sold the monopoly quantity at the monopoly price, the seller would like to sell a bit more, because the seller need not cut price on units already sold. Second, consumers will rationally anticipate such price cuts, and thus will hold out for future prices. Third, if the seller can change prices sufficiently fast, the path must go to marginal cost arbitrarily quickly, that is, the price will be marginal cost. This idea came to be known as the Coase conjecture.

Essentially the Coase conjecture holds that a monopolist compete with future incarnations of himself. Even though the most profitable course of action is to sell the monopoly quantity immediately, and then never sell again, the monopolist cannot resist selling more once the monopoly profit is earned. That is, subgame perfection condemns the monopolist to low profits.

The Commitment Solution

It is useful to consider the commitment solution as a benchmark, and to introduce notation. The seller's marginal cost is set to zero. Suppose time is discrete, with periods t=1,2,… Both the seller and the buyers discount each period at δ. Market demand is given by q, and is composed of a continuum of individuals.

The commitment solution involves a sequence of prices $p_1, p_2,\ldots$ This series of prices is non-decreasing without loss of generality, since no consumer will wait to buy at a higher price. A consumer with a value v will prefer time t to time t+1 if

(*) $\quad v - p_t > \delta(v - p_{t+1})$

These equations define a sequence of critical values v_t that make the buyer indifferent between purchasing at t and purchasing at t+1. (Note that the incentive constraint on buyers shows that, if a buyer with value v chooses to buy before time t, then all buyers with values exceeding v have also purchased by this time.)

$$v_t - p_t = \delta(v_t - p_{t+1})$$

This set of equations can be solved for p_t in terms of the critical values:

$$p_t = (1-\delta)v_t + \delta p_{t+q} = (1-\delta)v_t + \delta((1-\delta)v_{t+1} + \delta p_{t+2}) = \ldots$$
$$(1-\delta)\sum_{j=0}^{\infty} \delta^j v_{t+j}.$$

The monopolist sells $q(v_t)$-$q(v_{t-1})$ in period t, where v_0 is defined so that $q(v_0)$=0. The monopolist's profits are

$$\pi = \sum_{t=1}^{\infty} \delta^{t-1} p_t q_t = \sum_{t=1}^{\infty} \delta^{t-1} (1-\delta)\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} \right)(q(v_t) - q(v_{t-1}))$$

$$= (1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} \right) - \sum_{t=1}^{\infty} q(v_{t-1})\delta^{t-1}\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} \right)\right]$$

$$= (1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} \right) - \sum_{t=2}^{\infty} q(v_{t-1})\delta^{t-1}\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} \right)\right]$$

$$= (1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} \right) - \sum_{t=1}^{\infty} q(v_t)\delta^{t}\left(\sum_{j=0}^{\infty} \delta^j v_{t+1+j} \right)\right]$$

$$= (1-\delta)\left[\sum_{t=1}^{\infty} q(v_t)\delta^{t-1}\left(\sum_{j=0}^{\infty} \delta^j v_{t+j} - \delta \sum_{j=1}^{\infty} \delta^{j-1} v_{t+j} \right)\right]$$

$$= (1-\delta)\sum_{t=1}^{\infty} q(v_t)\delta^{t-1} v_t = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} q(v_t) v_t .$$

Thus, the optimum level of v_t is constant at the one-shot profit maximizing level, which returns the profits associated with a static monopoly. The ability to dynamically discriminate does not increase the ability of the monopolist to extract rents from the buyers.

How does the requirement that the monopolist play a subgame perfect strategy affect the monopolist's profits? To simplify the analysis, let demand be linear: $q(p)=1-p$. Consider a game that ends at time T. Let a_t refer to the highest value customer remaining in the population at the end of time t, so that the set of values remaining at the beginning of time t is uniformly distributed on $[0,a_{t-1}]$, and the quantity purchased at time t is $a_{t-1} - a_t$.

In the last period, the monopolist is a one-shot monopolist, and thus charges the price $p_T = ½\, a_T$ and earns profits $\pi_T = ¼ a_{T-1}^2$. This can be used as the basis of an induction to demonstrate that

$p_t = \lambda_t\, a_{t-1}$ and $\pi_t = \chi_t a_{t-1}^2$.

The last values satisfy $\lambda_T = ½$ and $\chi_T = ¼$.

The value a_t is determined by consumer indifference between buying at t and buying one period later, along with the beliefs that the monopolist will follow the equilibrium pricing pattern in the future, so that

$$a_t - p_t = \delta(a_t - p_{t+1}) = \delta(a_t - \lambda_{t+1} a_t),$$

or

$p_t = a_t\,(1 - \delta + \delta\lambda_{t+1})$.

Thus,

$$\pi_t = p_t(a_{t-1} - a_t) + \delta\pi_{t+1} = (1 - \delta + \delta\lambda_{t+1})a_t(a_{t-1} - a_t) + \delta\chi_{t+1}a_t^2$$

Maximizing this expression over a_t, we see that the firm chooses p_t to induces a_t satisfying

$$a_t = \frac{1 - \delta + \delta\lambda_{t+1}}{2(1 - \delta + \delta\lambda_{t+1} - \delta\chi_{t+1}})a_{t-1}.$$

Feeding this expression into π_t and simplifying gives

$$\pi_t = \frac{(1 - \delta + \delta\lambda_{t+1})^2}{4(1 - \delta + \delta\lambda_{t+1} - \delta\chi_{t+1}})a_{t-1}^2.$$

We have, at this point, verified the induction hypothesis – p_t is linear in a_{t-1} and π_t is quadratic, provided p_{t+1} is linear in a_t and π_{t+1} is quadratic

Since $\lambda_t\, a_{t-1} = p_t = a_t\,(1 - \delta + \delta\lambda_{t+1}) = \dfrac{(1 - \delta + \delta\lambda_{t+1})^2}{2(1 - \delta + \delta\lambda_{t+1} - \delta\chi_{t+1}}a_{t-1}$,

$$\lambda_t = \frac{(1 - \delta + \delta\lambda_{t+1})^2}{2(1 - \delta + \delta\lambda_{t+1} - \delta\chi_{t+1}}a_{t-1},$$

and,

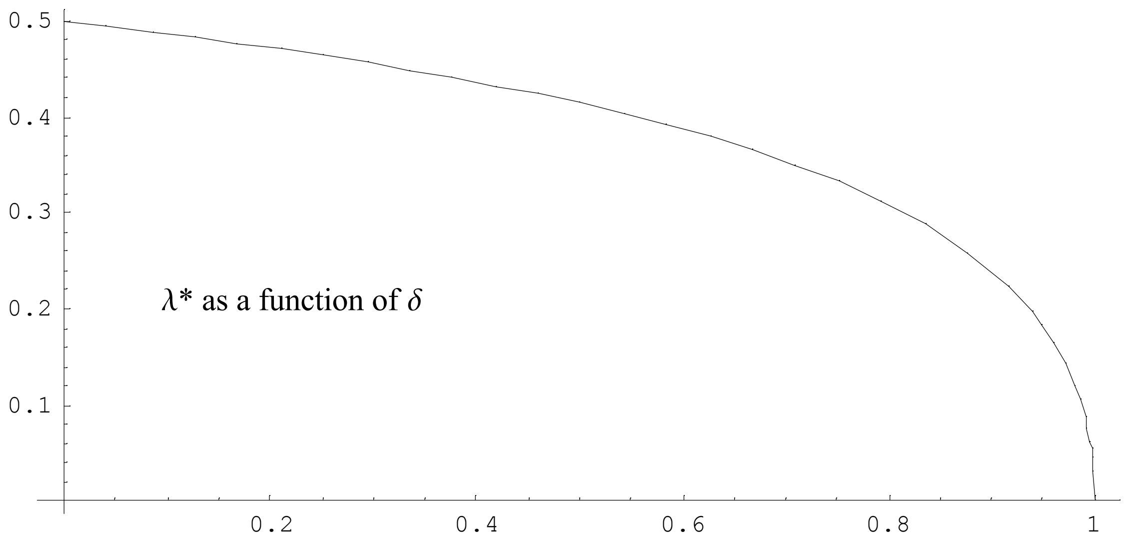
$$\chi_t = \frac{\pi_t}{a_{t-1}^2} = \frac{(1 - \delta + \delta\lambda_{t+1})^2}{4(1 - \delta + \delta\lambda_{t+1} - \delta\chi_{t+1}} = \frac{\lambda_t}{2}.$$

This permits the solution for λ_t in terms of λ_{t+1}.

$$\lambda_t = \frac{(1 - \delta + \delta\lambda_{t+1})^2}{2(1 - \delta + \tfrac{1}{2}\delta\lambda_{t+1})}a_{t-1}.$$

Define $f(\lambda) = \dfrac{(1 - \delta + \delta\lambda)^2}{2(1 - \delta + \tfrac{1}{2}\delta\lambda)}$, so that $\lambda_t = f(\lambda_{t+1})$. $f(0) = \tfrac{1}{2}\,(1-\delta)$ and $f(1) = 1/(2-\delta)$. It is readily shown that f is increasing and strictly convex for $\delta \in (0,1)$. There is a unique fixed point for f, which occurs at

$$\lambda^* = \frac{\sqrt{1 - \delta} - (1 - \delta)}{\delta} < \tfrac{1}{2}.$$



Since λ_T=½, the sequence λ_t is increasing in t to ½. For games with very large values of T, λ_1 is very close to λ^*. The opening price offered by the monopolist is λ_1, because a_0=1. The Coase conjecture amounts to the claim that, when the monopolist can cut prices very rapidly, the opening price is close to marginal cost, which was set to zero. The ability to cut prices very rapidly corresponds to a large discount factor – little discounting goes on between each pricing period. The Coase conjecture is in fact true, because

$$\lim_{\delta\to 1}\lambda^* = \lim_{\delta\to 1}\frac{\sqrt{1-\delta}-(1-\delta)}{\delta} \to 0.$$

This equilibrium is representative of all equilibria in the "gap" case, which is the case that arises when consumer valuation exceeds marginal cost by some positive amount for all consumers. In this case, price converges to the minimum consumer valuation, rather than to marginal cost. The "gap" case is not empirically relevant. The gap case corresponds to backward-induction equilibria because in fact the monopolist will sell all its output in finite time in equilibrium.

In addition, Gul, Sonnenschein and Wilson show that equilibria with stationary strategies also have the Coase property.

In contrast, Ausubel and Deneckere show that Coasian equilibria can be used to sustain other equilibria in which the monopolist makes positive profits. Consider a decreasing price path p_t and suppose that consumers hold the conjecture that the monopolist will follow that path in equilibrium, and if that consumers see any price charged other than a price from the path, consumers believe that the monopolist will play the Coase path. The Coase path produces very low profits, so the threat of such beliefs is sufficient to sustain a large set of equilibria. These equilibria are a bit weird, since consumers beliefs help support prices that sustain high profits, but are sequential, if nonstationary, equilibria nevertheless. They are reasonable in that they predict a declining path of prices, with any lower prices suggesting even faster future decreases in prices.

Means of Mitigating the Coase Problem

Do we really believe that a durable goods monopolist prices at marginal cost? There are several strategies a monopolist might employ to prevent his tendency to *compete with himself*.

1. *Other equilibria* don't have this property, but stationary (history independent) ones typically do.

2. *Leasing vs. selling*: A monopolist that leases the durable good no longer has the incentive to cut price to bring in new consumers, because he must cut the price to the old ones as well, who are, after all, leasing and can always turn it in and release at the lower price.

3. *Return policy or money back guarantee*: Suppose the monopolist allows one to return the good for the full purchase price, to be credited against future purchases. Then consumers need not wait for prices to fall - they've been given a guaranteed low price if prices do fall. This, of course, provides the same kind of disincentive to lower prices as leasing. Used by department stores on calculators in the early 1970s.

4. *Destroy the production facility*: used for limited edition items on occasion.

5. *Make remaining in the market expensive*: the profits in future periods are, of course, a decreasing function of time; the monopolist is cutting price and the high value consumers have left. Therefore, eventually, the monopolist will choose to exit [Singer almost exited the sewing machine industry on this logic, but stayed in after its announcement that it would leave because of public relations, or so they said]. However, this is mitigated by the entry (birth) of new consumers. But new consumers also mean that the monopolist has less incentive to cut prices, because he also cuts the price to the high value consumers. This can cause price cycles in some models, with the monopolist occasionally cutting price to grab the low value consumers who are accumulating.

6. *Keep the marginal cost secret*: If buyers aren't informed about the true marginal cost, their expectations will be influenced by the prices the monopolist charges, and the monopolist may not cut price, so as to convince consumers that he has high cost.

7. *Planned Obsolescence*: Note that a good that breaks down after one period - i.e. a nondurable good - is equivalent to leasing rather than selling. Thus, a monopolist who must sell has an incentive to reduce the durability of his good, that is, plan obsolescence. The standard example is a textbook manufacturer who frequently introduces new editions, to kill off the used book market, and is therefore producing a nondurable good when a durable good is feasible and often optimal.

8. *Capacity Choice*