Handout #4: Price Dispersion

George Stigler's 1961 "The Economics of Information" begins with:

> One should hardly have to tell academicians that information is a valuable resource: knowledge *is* power. And yet it occupies a slum dwelling in the town of economics.

Stigler documents the existence of price dispersion empirically, and proceeds to sketch various theories that might be applicable to price dispersion. Stigler is careful to distinguish price dispersion, which results from imperfect information on the part of consumers, from price discrimination, which involves distinct prices for different types of consumer preferences.

There are various reasons that price dispersion might arise. Clearly some consumers must be willing to pay higher prices than others, that is, distinct consumers have distinct *reservation* prices. (Note that a reserve price – the minimum bid in an auction – is a very different concept.) Differences in reservation prices can arise because of differences in knowledge – some consumers have access to price lists, others don't – or search costs. If some consumers have lower search costs, then these consumers will search for lower prices.

Models based on search costs usually require differences in the firms as well as differences in search costs on the part of buyers. The reason is that, with differences in consumer search, identical firms will tend to respond in an identical fashion to the generated demand, and the price dispersion is degenerate. An extreme example of this phenomenon is the *Diamond Paradox*. (Diamond, Peter, A Model of Price Adjustment, *Journal of Economic Theory,* 1971, 156-68.) Suppose that there is global minimum to search costs γ, that is, all consumers have search costs in excess of $\gamma>0$. Search costs arise per store – each store sampled costs γ or more. Then all firms charge the monopoly price. The proof is straightforward. Let L be the lower bound on the distribution of prices and suppose $L < M$, the monopoly price. Then charging $\text{Min}\{L + \frac{1}{2}\gamma, M\}$ is strictly better than charging any lower price, because no consumer will reject the price $L + \frac{1}{2}\gamma$ that will accept L, because the cost of obtaining an additional price, even if it is certain to be L, is at least γ! Thus, even if all consumers have a very low but positive cost of search, the equilibrium involves all firms charging the monopoly price. What is paradoxical about this result is that the equilibrium is discontinuous in the search cost – the equilibria with zero search costs (price equals marginal cost) and positive search costs are very different.

The literature in the 1970s focused on models with varying search and production costs as a means of generating price dispersion. (See Carlson and McAfee, *J Pol. Econ.* 1983 for an example and a list of references.) The more modern and economical approach involves generating price dispersion from identical firms via randomization. This approach is intellectually more satisfying because we don't need a story for why the firms have different costs; moreover, applications involving entry of identical firms become possible. Moreover, it is possible to generate the consumer informational asymmetries endogenously, as Varian (*AER*, 1980) does.

The Butters Model

Suppose n firms send advertisements to a proportion α of the population with price offers. Each consumer chooses the price offer that is lowest. It will not be an equilibrium for each firm to send out the same price offer - it would pay to undercut it. So we will look for a mixed strategy - each firm sends out a random price offer. Let $F(p)$ be the probability that a price offer is not more than p.

A firm's profits per consumer are

$$\pi(p) = (p\text{-}c)[1\text{-}\alpha + \alpha(1\text{-}F(p))]^{n\text{-}1}$$

since it beats another firm if the consumer doesn't receive an offer from that firm (1-α) or if the offer received from that firm has a price in excess of p. Let v be the maximum the consumer will pay. Someone has to send offers of v - because if the maximum offer were less than that, it would pay to send out higher offers [the highest offer is only accepted if no other offer is received]. Thus

$$\pi(p) = \pi(v) = (v\text{-}c)[1\text{-}\alpha]^{n\text{-}1}$$

Therefore

$$1-\alpha F(p) = 1-\alpha+\alpha(1-F(p)) = (1-\alpha)\left(\frac{v-c}{p-c}\right)^{\frac{1}{n-1}}.$$

or

$$F(p) = \frac{1}{\alpha}\left[1-(1-\alpha)\left(\frac{v\text{ - }c}{p\text{ - }c}\right)^{\frac{1}{n-1}}\right].$$

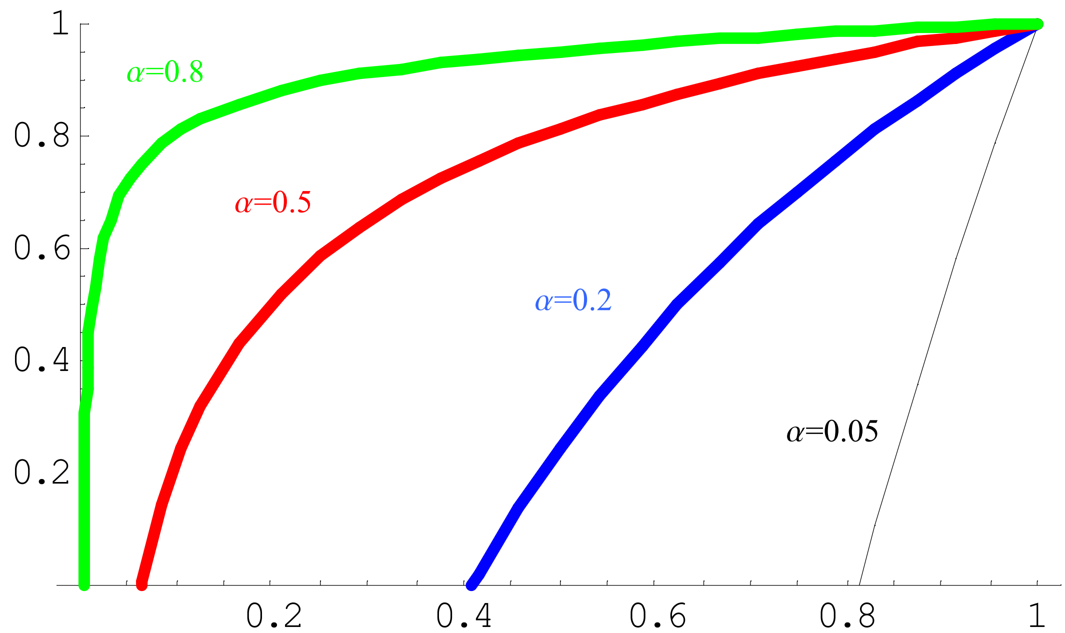
The lowest price, p_o, is the price such that $F(p_o)$=0, or $p_o = c + (1\text{-}\alpha)^{n\text{-}1}(v\text{-}c)$.

The distribution of prices is Prob(offer$\leq p$) = 1 - $[1\text{-}\alpha + \alpha(1\text{-}F(p)]^{n\text{-}1}$. Thus the expected price minus cost is

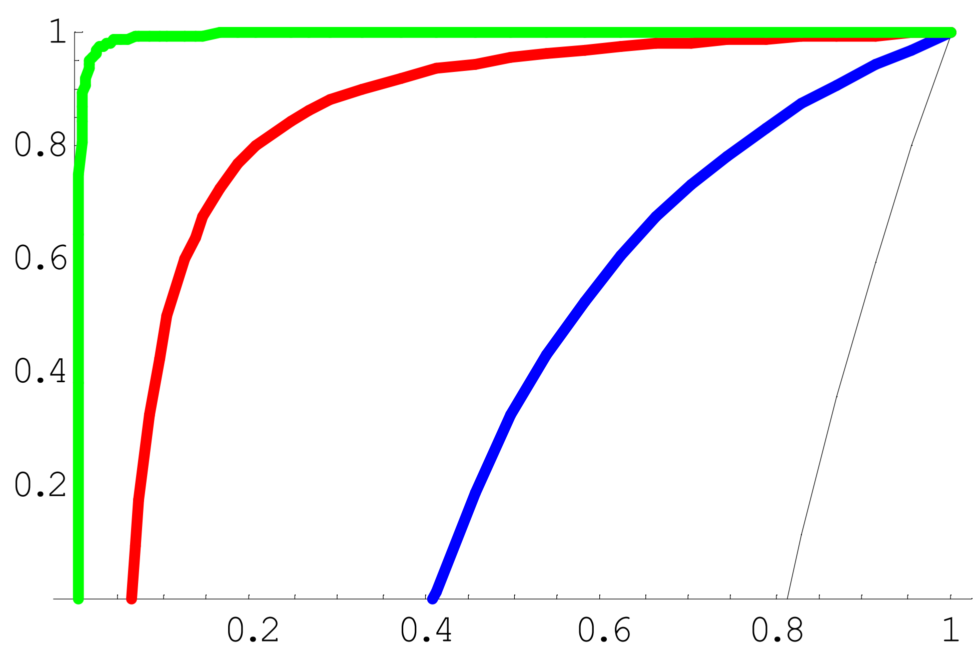
$$E[p-c] = \int_{p_0}^{v}(p-c)n(1-\alpha F(p))^{n-1}\alpha f(p)dp = \int_{p_0}^{v}(p-c)n\pi\alpha f(p)dp = n\pi\alpha = n\alpha(v-c)(1-\alpha)^{n-1}.$$

This term doesn't account for the probability that no offer is received. Thus, of the people who purchase, the expected price is the above term divided by $1\text{-}(1\text{-}\alpha)^n$. This corresponds to consumers who fail to purchase searching again.

To look at merger in the Butters model, we would have to consider asymmetric α - a much more difficult problem. However, the Butters model produces applicable imperfect price competition in some circumstances, and also produces random prices - a model of sales.



The probability distribution of offered prices for various values of α, and the best price realized by consumers. c=0 and n=5.



Note that advertising in the Butters model is *informative*, telling consumers where they can get a low price. Advertising in the *burning money* or market for lemons environment serves only the purpose of telling consumers "If you believe my product to be superior as a result of this ad, then it is profitable for me to burn money, and it would not be profitable, were my product inferior, so you should believe me".

The solution to the Butters model is equivalent to the auction model in which bidders have independently distributed values in {0,1}, and α is the probability that a given bidder has value equal to 1. Such a bidder uses a mixed strategy F, and the mixed strategy produces profits

The Butters model can be varied in a straightforward way by letting buyers either know one price (chosen randomly) or all the prices, which is accomplished in Varian's AER paper. Adding a quantity choice that depends on price introduces no complexity This gives a profit function

$$\pi(p) = (p-c)q(p)\left(\beta/_n + (1-\beta)(1-F(p))^{n-1}\right)$$

Varian goes on to endogenize the number of consumers who choose to be informed given a cost of being informed, an important development because it makes the entire model self-contained and rational.

As posited, the price dispersed models are beautiful models that are not useful for industrial organization applications. To make them more useful, it is necessary to endogenize the access of the firms to consumers, that is, endogenize the value of α. This is done in

McAfee, R.P. "Endogenous Availability, Cartels and Merger in an Equilibrium Price Dispersion," *Journal of Economic Theory 62*, no. 1, February 1994, 24-47.

Asymmetric Price Dispersion

Firms have availability rate α_i, ranked from largest to smallest, so that $\alpha_1 \geq \alpha_2 \geq \ldots$

Let $R(p)=(p-mc)q(p)$ and p^m maximize R. Then with a substantial amount of work, one can show that there is an equilibrium, that firm 1 has a mass point at p^m if $\alpha_1 > \alpha_2$, that the firms with lower availability randomize over intervals with lower prices, and finally that profits per unit of availability are the same, but that the largest firm enters asymmetrically into the profit equation.

$$\pi_i = \alpha_i R(p^m) \prod_{j \neq 1} (1-\alpha_j).$$

Introduce a cost $c(\alpha)$ of availability. Given the multiplicative nature of probabilities, the size of a scale economy is given by the cost saving associated with combining the operations of two entities, which is $c(\alpha)+c(\beta)-c(1-(1-\alpha)(1-\beta))$. Consequently, availability has increasing returns to scale whenever $c(\alpha)+c(\beta)-c(1-(1-\alpha)(1-\beta))$ is increasing in α or β. It turns out that increasing returns to scale are equivalent $(1-\alpha)c'(\alpha)$ being decreasing in α. Constant returns to scale involve $(1-\alpha)c'(\alpha)$ being constant, which implies $c(\alpha)=-\theta\log(\alpha)$.

Theorem: There is a pure strategy equilibrium, which involves $\alpha_1 > \alpha_2 = \alpha_3 = \ldots = \alpha_n$. If there are increasing return to scale, $\alpha_1 > 2\alpha_2$.

The closed form solution to the price dispersed equilibrium makes it a natural vehicle for industrial organization theory.